On the numerical scheme employed in gyrotron interaction simulations

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Abstract
We report on the influence of the numerical scheme employed in gyrotron interaction simulations. Results obtained with the Crank-Nicolson scheme are compared with those obtained with the Backward Time - Centred Space (BTCS) fully implicit scheme. We present realistic cases where, for discretisation parameters in the range usually used in gyrotron simulations, the results can be very different. Hence, the numerical scheme used can be responsible for obscuring the underlying physics if its convergence is not tested carefully.

1. Introduction
• The European Gyrotron Consortium (EGYC) is developing a 170 GHz gyrotron in support of EU contribution to ITER [1][2].  
• Fast simulations of the beam-wave interaction are essential for gyrotron design and support to the experiments.  
• There are four European codes, available at EGYC, capable of fast simulations: EURIDICE [4], SELFT [5], COAXIAL [6], and TWANG [7].  
• The results of the codes are in agreement in the majority of cases. Some discrepancies that appear are understood and attributed to the difference in the interaction models.  
• However, in some situations of interest, the results of COAXIAL have been significantly different from those of the other codes. This triggered an investigation which indicated that this discrepancy was a consequence of the difference in the numerical schemes. COAXIAL uses a scheme which is first-order in time, whereas the rest of the codes use schemes that are second-order in time.  
• This indication was confirmed after EURIDICE was modified to be capable of using either of the two schemes and pertinent investigations, presented in this paper, were performed.

2. Model and Numerical Schemes
Interaction model for reference
\[ \frac{dF}{dt} + \nabla \cdot \left[ \left( \frac{\epsilon_{\perp} - 1}{\epsilon_{\parallel}} \right) \nabla F \right] = \sum_i \left( \delta f_i \right) + \sum_i \left( \delta p_i \right) \]

Finite difference schemes for the field equation
\[ \frac{\Delta F}{\Delta t} = \frac{1}{2} \left( \Delta F_{n+1} - \Delta F_n \right) + \frac{1}{2} \left( \Delta F_{n+1} - \Delta F_{n-1} \right) \]

1. Backward-Time Centred-space fully implicit scheme (BTCS)
\[ F_{n+1} = F_n - \frac{\Delta t}{\mu} \nabla \cdot \left( \frac{\epsilon_{\perp} - 1}{\epsilon_{\parallel}} \nabla F_n \right) \]

2. Crank-Nicolson scheme (C-N)
\[ \frac{F_{n+1} - F_n}{\Delta t} = \frac{1}{2} \left( \frac{\epsilon_{\perp} - 1}{\epsilon_{\parallel}} \nabla \cdot \nabla F_{n+1} + \frac{\epsilon_{\perp} - 1}{\epsilon_{\parallel}} \nabla \cdot \nabla F_n \right) \]

The BTCS scheme needs three orders of magnitude smaller time-step \( \Delta t \) to agree with the results of the Crank-Nicolson scheme.

Note: Dynamic ACI is an effect that is currently under extensive analysis regarding both the experimental findings and the capability of the present interaction models and codes to describe it correctly [9][10]. Discussion on the validity of the models and codes in treating dynamic ACI is beyond the scope of this paper.

3. Numerical Results
The influence of the numerical scheme on the simulations was investigated by EURIDICE in two realistic cases, performing convergence studies with the time-step \( \Delta t \) as parameter.

3.1 Dynamic After-Cavity Interaction
When the simulated interaction region incorporates also the non linear upperator after the cavity, dynamic After-Cavity Interaction (ACI) has been observed in some cases [9].

3.2 Test case: 170 GHz, 1 MW gyrotron operating in the TE\(_{22}\) mode, EU fast-back solution for ITER [1].

3.3 Numerical Results
The results of the codes are in agreement in the majority of cases. Some discrepancies that appear are understood and attributed to the difference in the interaction models. However, in some situations of interest, the results of COAXIAL have been significantly different from those of the other codes. This triggered an investigation which indicated that this discrepancy was a consequence of the difference in the numerical schemes. COAXIAL uses a scheme which is first-order in time, whereas the rest of the codes use schemes that are second-order in time.

This indication was confirmed after EURIDICE was modified to be capable of using either of the two schemes and pertinent investigations, presented in this paper, were performed.

4. Discussion
• We have compared results of simulations of the beam-wave interaction in gyrotrons in two realistic cases using two different numerical schemes: The BTCS fully implicit scheme and the Crank-Nicolson (C-N).
• It was shown that BTCS needs about three orders of magnitude smaller time-step in order to converge to the results of C-N. Otherwise the BTCS results can be even qualitatively different. This makes the BTCS totally inefficient in the simulated cases, as the required simulation time becomes very large.
• Such behaviour may be expected, since BTCS is 1\textsuperscript{st} order in time, whereas C-N is 2\textsuperscript{nd} order in time. In the vast majority of cases in practice, however, such behaviour is not encountered. Both schemes agree on their results using a time-step of the order of \( 10^{-5} \text{ s} \), which is the one usually used in interaction simulations of mm-wave gyrotrons. This has created a confidence in both schemes.
• Our results indicate that there are cases where the above confidence needs to be reassessed, as far as the BTCS scheme is concerned.

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