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PERIODIC STRUCTURES IN PLASMA FOR ELECTRON ACCELERATION AND FREE ELECTRON LASERS

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1. Introduction

A free-electron laser (FEL) is a device based on the free longitudinal motion of relativistic electrons and on the small transverse oscillations in a spatially periodic electric or magnetic field (undulator). After the first demonstration of lasing of a relativistic electron beam in an undulator [1], FEL physics has achieved high output power in a very wide wavelength range, from mm down to soft UV (see review [2]). One of the main features of FELs is their tunability due to the dependence of the radiation wavelength on beam energy and undulator field. Not all problems of FELs are well understood yet, but FELs are working already as user facilities in the infrared [3,4] and are considered as a most promising tool for tokamak plasma heating [5,6].

The extension of FELs to the VUV and X-ray regions would be extremely useful for many applications [7]. However, the basic FEL scheme with a magnetostatic undulator encounters two difficulties. Firstly, the present state of the art of undulator construction provides undulators with a period above 2 cm and there is no perspective to make it significantly shorter with the necessary amplitude of magnetic fields of kG order. It means that for X-ray production one needs an electron beam energy near 1 GeV or more. This leads to big and expensive accelerators. Secondly, since the FEL gain is very low for X-rays and no efficient reflecting mirrors exist, sufficient amplification has to be reached during one passage of the beam which can only be accomplished by increasing the number of periods. With the shortest possible period of 2 cm the total length should be several tens of meters with very high requirements on the precision of undulator construction. Thus, the radiator would also be very big and expensive. The conclusion is that the traditional FEL scheme will never yield a compact X-ray source.

Linear accelerator and FEL physics have many features in common. Beam-wave interaction in a FEL can be described by the same mathematical formalism as used for linacs [8,9]. The origin of this similarity lies in the fact that the resonance between free electrons and radiation occurs at velocities below the speed of light. The mechanisms of the formation of such a wave are different for linacs and FELs, but particles experience the same longitudinal bunching which can be represented in terms of energy - phase variables with the help of the pendulum equation. Equations that describe FELs with a tapered undulator [10] are almost identical to the equations for resonant linacs.

The typical acceleration rate for linacs does not exceed 10 MeV/m [11]. Therefore one cannot relay on traditional methods of acceleration to obtain a compact accelerator for high energy electron beams.

It is known that a plasma is a medium where intensive waves can be excited in the presence of different configurations of magnetic and electric fields or electromagnetic waves. Some of them fields. From this point of view plasmas are extremely attractive for acceleration of electrons and for the production of short wavelength radiation by relativistic beams. The aim of this report is to review the possibilities for utilization of plasmas for both purposes.

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2. REVIEW OF PLASMA ACCELERATION CONCEPTS

The longitudinal (Langmuir) plasma wave (LPW) is the most convenient plasma instability that can be used for electron acceleration if it travels with a relativistic velocity. This way of acceleration has two obvious advantages in comparison with the usual electromagnetic wave acceleration technique: i) particles are accelerated directly by the longitudinal field without conversion of transverse fields into longitudinal ones; ii) a high amplitude of accelerating field can be reached. The main point for this plasma acceleration technique is the method of excitation of a travelling LPW.

The most promising concept where the LPW is driven linearly is the beat-wave accelerator (BWA) [1]. The basic idea of the BWA is illustrated in Fig. 1. Two laser beams with slightly shifted frequencies are focused into a plasma tube. A fully ionized, high-density plasma in the range 10^{16}-10^{18} cm^{-3} is produced by either θ- or z-pinches, RF waves or by multiphoton ionization. Coils for producing a confining magnetic field may be necessary.

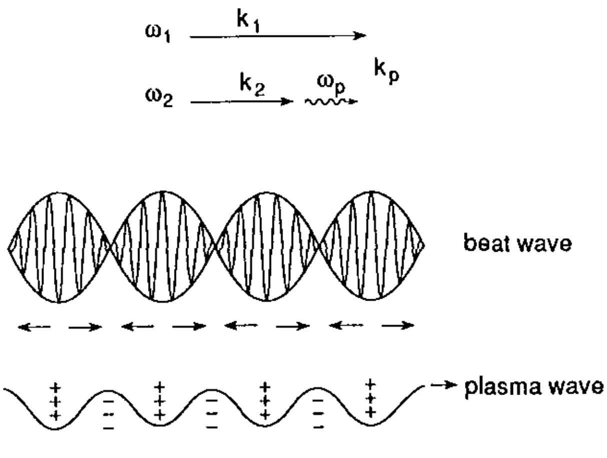

Fig. 1. The beat-wave acceleration scheme.

Transverse electromagnetic waves in plasmas have a group velocity which is less than the speed of light. If the plasma frequency is resonant with the beat frequency of the two lasers, a longitudinal plasma wave with large amplitude is excited under the action of the ponderomotive force produced by the transverse laser fields. Therefore, the BWA is based on the resonance conditions

$$\omega_p = \omega_1 - \omega_2, \ \mathbf{k}_p = \mathbf{k}_1 - \mathbf{k}_2 , \tag{2.1}$$

where $\omega_{1,2}$, $\mathbf{k}_{1,2}$ are the frequencies and wavenumbers for the first and second laser beam, respectively, and $\omega_p^2 = 4\pi n_p e^2/m$ is the plasma frequency. In the limit $\omega_p << \omega_{1,2}$ the phase velocity of the LPW approaches the group velocity of light

$$v_{ph} \cong v_g = c(1 - \omega_p^2/\omega_1^2)^{1/2} . \tag{2.2}$$

If bunches of electrons from a linac are synchronized with the laser pulses, those electrons which are trapped in the right phase in the LPW will be accelerated by the wave's large longitudinal electric field. The amplitude of this field E_{max} is determined by Poisson's equation and the acceleration rate equals to $dE_a/dz = eE_{max}$. Because the average oscillation amplitude of the plasma electrons is of the order of the plasma wavelength, λ_p, the acceleration rate is restricted by the following relation

$$dE_a/dz < mc^2/\lambda_p \,. \qquad (2.3)$$

Taking into account that the plasma wavelength is equal to $\lambda_p \cong 0.3\, n_p^{-1/2} \cdot 10^7$ one obtains that for $n_p \cong 10^{16}$ cm^{-3} the acceleration rate is as high as 1.5 GeV/m, which is two orders of magnitude higher than the maximum value for traditional RF techniques.

In the laboratory frame the length over which acceleration takes place cannot exceed the value

$$L = 2\gamma_{ph}^2 \lambda_p \,. \qquad (2.4)$$

The total energy acquired by an electron, $E_{tot} = L dE_a/dz$, is limited to

$$E_{tot} < 2mc^2\gamma_{ph}^2, \qquad (2.5)$$

where $\gamma_{ph} = (1 - \omega_p^2/\omega_1^2)^{-1/2}$ for BWA.

Relations (2.3)-(2.5) are valid for any method of excitation of the linear LPW.

Two other methods were proposed for driving the linear LPW: by a short electron pulse and by a short laser pulse (see [2,3]). In both cases the length of the pulse should be shorter than the plasma wavelength. In addition, the pulse should have a high intensity and a special form [2]. These requirements make the practical realization of these concepts very difficult.

Of the several experiments that have been done, the most impressive results have been obtained recently in UCLA [4]. Their CO_2 laser system produced a two-frequency laser beam with 60 J at wavelength $\lambda_1 = 10.59$ µm and 10 J at $\lambda_2 = 10.29$ µm. The laser radiation was focused to a nearly diffraction limited spot of 300 µm diameter, resulting in peak pump parameters $p_{1,2} =$ 0.17 and 0.07. The plasma, produced by tunnel ionization of hydrogen gas with the pressure in the range 110-200 mTorr, had a density near 10^{16} cm^{-3}. CCD camera images showed that the plasma extended over more than 20 mm and was fully ionized over more that 12 mm. Injected 2.1 Mev electrons gained the energy up to 9.1 MeV, thus the acceleration rate was 0.7 GeV/m, which is very close to the simple theoretical estimate (2.3).

The resonant excitation of the LPW which is the basic mechanism of the BWA concept requires the strict fulfilment of conditions (2.1). As any resonant method, it is very effective in initializing the instability, but any deviation from (2.1) will destroy the whole process. This implies two obvious restrictions on the BWA.

Firstly, in order to reach an electron energy near 1 GeV, the length of the plasma column should be 1 m according to (2.4) . Therefore, the resonance condition requires the plasma to be homogeneous along the whole length. Any spatial dependence $n_p(z) = n_{po} + \delta n$ will cause a violation of conditions (2.1). In addition, due to the dispersion relation for the LPW

$$\omega_p^2 = \omega_{po}^2 \left(1 + 3 \frac{k_p^2 v_{Tp}^2}{\omega_{po}^2} \right), \qquad (2.6)$$

where v_{Tp} is the thermal velocity of plasma electrons, thermal motion of plasma electrons can also break down the resonance. Hence, the plasma should be cold enough and sufficiently homogeneous to have a nearly constant value of ω_p.

Secondly, the maximum acceleration rate and total energy are generally reached for values of the plasma modulation degree $\varepsilon = \delta n_p/n_p$ near unity. To obtain $\varepsilon \sim 1$ the pump parameter of laser pulses, $p = eE_{1,2}/mc\omega_{1,2}$, must also be near unity, where $E_{1,2}$ are the amplitudes of the laser field. However, at high levels of radiation power injected into the plasma, the plasma frequency is changed due to nonlinear effects, $\omega_p = \omega_{po} + \alpha E^2$, which leads to the violation of conditions (2.1). Hence, the LPW exists only in the linear regime, so that the velocity of the plasma electrons may not be too close to the speed of light and the acceleration rate is restricted by the rest energy of electron (2.3).

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3. REVIEW OF PLASMA FELs

FELs based on the induced radiation from an electron beam passing through a plasma have a much shorter history than the BWA. Two ideas about how to produce an undulator in a plasma were suggested. In both schemes electrons experience periodic transverse oscillations. The interaction mechanism can be described by a formalism that is similar to the one used to describe FELs with magnetostatic undulators.

The first scheme is the ion channel FEL (ICFEL). The ion-focused regime of transport of relativistic electron beams through plasmas was proposed in [1]. This regime is characterized by $n_b > n_p$, where n_b is the density of electron beam. Plasma electrons are ejected from the channel by the space charge of the entering beam. The radius of the ion channel is $b \sim r_b(n_b/n_p)^{1/2}$, where r_b is the radius of the beam, so that a wide ion channel is formed (Fig. 2) if n_b significantly exceeds n_p. The electrostatic focusing in such a channel is linear.

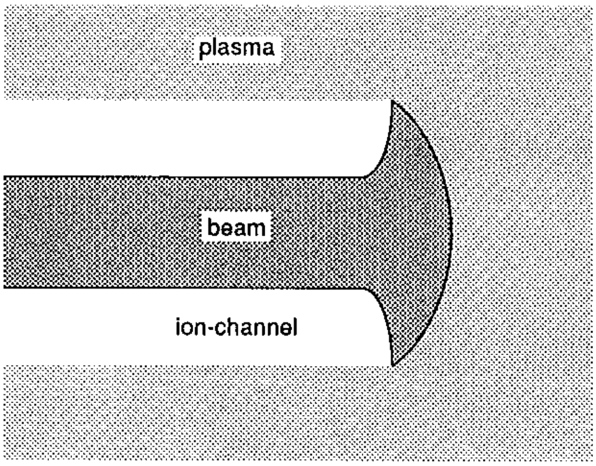

Fig. 2. An ion channel in a plasma.

As shown in Refs [2,3] this configuration is unstable against electromagnetic perturbations. The origin of this instability lies in transverse betatron oscillations of electrons in the electrostatic potential of the ion channel

$$U = -\frac{m\omega_p^2}{4e}\left(x^2 + y^2\right), \tag{3.1}$$

where x and y are transverse coordinates. The electron beam propagates in the z-direction.

The transverse motion of electrons is periodic in the potential (3.1)

$$\frac{d^2r}{dz^2} + k_\beta^2 r = 0, \tag{3.2}$$

where the betatron wavenumber is $k_\beta^2 = \omega_p^2/2\gamma c$, γ being the relativistic factor of the beam. Because of the transverse oscillations of electrons in the presence of an electromagnetic wave a ponderomotive wave is created with phase velocity below the speed of light, $v_{ph} = \omega_s/(k_s+k_\beta)$, where ω_s is the frequency of the e.m. wave. Electrons can be in resonance with the electromagnetic wave and radiate coherently. After averaging over the betatron period, the equations of motion take a form that is similar to the usual FEL equations (see Ref. [10] of the introduction). In the expression for the FEL gain (see Section 5) the pump parameter p has to be replaced by ε_n/r_b, where ε_n is the normalized beam emittance. This parameter plays the role of the undulator parameter a_w in FELs with magnetostatic undulators.

An experiment has been already done in KEK (Japan) in which an 800 keV, 800 A electron beam propagates through a column of laser ionized diethylaniline gas [4]. An output power in the several hundred kW range at 9.4 GHz has been observed. In principle, the frequency of the radiation grows with the beam energy like $\omega_s \sim \gamma^{3/2}$ so that much shorter wavelengths can be generated with higher energy beams.

The ICFEL is based on the periodic transverse motion of electrons in the absence of any a priori periodic structure in the plasma.

The second scheme is the ion ripple (IRFEL). In this system a periodic density modulation is created [5,6]. This density modulation can be produced by two methods: i) a sound wave can be used to modulate the density of a neutral gas with successive fast ionization by a laser pulse; ii) an ion acoustic wave can be excited in a neutral plasma. Because the ion acoustic speed is much less than the relativistic electron velocity, the ion density modulation will be seen by an electron as a stationary undulator. This density modulation is $n_i = n_{io}(1 + \varepsilon \sin k_i z)$, where n_{io} is the initial ion density, k_i is the wavenumber of the plasma ripple, and ε is the modulation degree. In order to produce radiation from the neutral plasma ripple, the relativistic electron beam should be injected into the plasma ripple at an angle θ (Fig. 3). As long as the beam density n_b is equal to or higher than the plasma density n_p, the plasma electrons will be expelled from the path of the beam in response to the space charge of the beam electrons [6]. This means that the IRFEL is a combination of an ion ripple and an ion channel.

The electric field seen by the beam can be expressed as

$$\mathbf{E}_i = \frac{4\pi n_{io} e}{k_i} \varepsilon \cos(z k_i \cos\theta)\left(\mathbf{x}\sin\theta - \mathbf{z}\cos\theta\right) . \qquad (3.3)$$

This field provides transverse oscillations of the electron beam that are periodic in z. The pump parameter in this case is defined by the multiplication factor $\varepsilon\sin\theta$. It determines the growth rate of the electromagnetic instability.

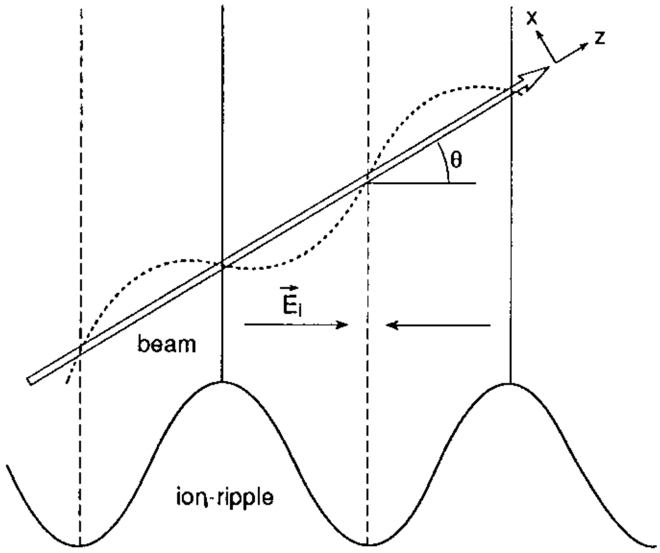

Fig. 3. A relativistic electron beam moving across a plasma ripple with an angle θ. The dashed curve is the actual beam trajectory.

In case of a deep modulation with $\varepsilon \cong 0.5$ of a background of initial density $n_{io} = 10^{15}$- 10^{16} cm^{-3} and electron energy ~50 MeV, the radiation frequencies are in the VUV or soft X-ray range.

Below we will discuss the prospects of application of the ICFEL and IRFEL concepts to the short wavelength range.

The ion channel is formed when the condition $n_b > n_p/2$ is fulfilled. The "undulator" period of the ICFEL is defined by the relation $\lambda_\beta = \lambda_p\sqrt{2\gamma}$ (see (3.2). Hence, to reach the value $\lambda_p = 1$ mm the current density of the electron beam has to be extremely high, j_b = 10 MA/cm^2. Moreover, in the case of operation with even a moderate beam energy of 50-100 MeV, the period increases significantly to $\lambda_\beta \geq 2$ cm. This is of the order which is already available with usual magnetostatic undulators. Besides, a very long ion channel of dozens of meters length should be created in a sufficiently dense plasma. This means that a number of effects like beam erosion and wake field action on the beam along the ion channel - plasma boundary become important. In addition, for a fixed current the creation of an ion channel in a dense plasma electron requires beams with very small radii. However, the pump parameter $p = k_\beta r_b$, which determines the gain, is proportional to r_b and should be near unity. Hence, the beam radius should not be very small what contradicts the previous requirement. Therefore, the ICFEL with a not too relativistic electron beam can be a perspective source of cm and probably mm-wavelength radiation .

The ICFEL is based on the same mechanism which governs the channeling radiation from electrons in a crystal [7]. The period of transverse oscillations in a planar crystal channel is defined by the relation $\lambda_c = d_p\sqrt{2\gamma}$, where d_p is the distance between crystallographic planes. Hence, the period is always very small and even an electron beam of several MeV electron beam emits X-ray radiation. The problem with the induced radiation is that due to multiple scattering electrons cannot perform many oscillations along its travel through the channel [8,9]. The typical values of the dechanneling length correspond to the region 1-10 μm. The radiation at some angle to the direction of electron motion can be used to increase the amplification length. In this case for the radiation wavelength λ_s = 200 nm the gain is roughly equal, $G \approx 10^{-7} j_b$ cm^{-1} [8,9]. The crystal dimension can be about 10 cm, hence, the required beam density must also be very high, $j_b > 10$ MA/cm^2.

The main disadvantage of the IRFEL is the lack of self-consistency. The optimum undulator structure should have simultaneously a minimal period and a high amplitude of the transverse field . In the IRFEL, the condition for a minimal period contradicts the maximal field condition, $\lambda_w = \lambda_i/\cos\theta$, $E_{tr} = E_i \sin\theta$. This is due to the fact that the longitudinal ripple fields are used for bending of the electron beam in the transverse direction. Besides, the problems of a long ion channel creation overlap with the problem of creation of the deep ion density modulation with the large number of periods in sufficiently dense plasmas.

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4. ACCELERATION OF ELECTRONS BY NONLINEAR PLASMA WAVES

From the general physics' point of view it is preferable to excite the LPW by the action of a longitudinal force. In the BWA the excitation of the LPW is implemented due to the conversion of the transverse electromagnetic fields into longitudinal fields through the action of the ponderomotive force. It will be argued that nonlinear longitudinal oscillations can be excited directly by the beam-plasma interaction. In this section we consider the excitation of the nonlinear LPW (NLPW) by a powerful electron beam (Fig. 4). NLPW provides a higher acceleration rate and a higher total energy of trailing electrons. Resonance conditions do not play a role in this mechanism.

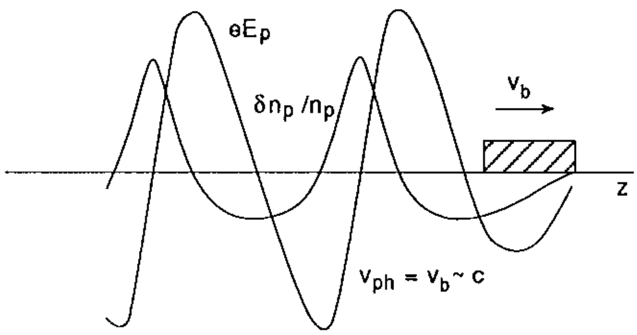

Fig. 4. A relativistic electron pulse propagating in a plasma excites plasma wakefield and electron density modulation.

Let an electron pulse travel through the plasma. We consider the plasma ions immobile and we do not take into account the back influence of the excited field on the pulse. Let the system be one-dimensional without magnetic fields. The Maxwell equations are in this case

$$\mathrm{div}\mathbf{E} = 4\pi\rho \,, \qquad \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j} = 0 \,. \tag{4.1}$$

The driving electron pulse has a relativistic velocity v_b and we will seek a solution of (4.1) in the form of travelling waves $E = E(z - v_b t)$. Introducing the new variable $\xi = z - v_b t$, we will have

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \xi} \,, \qquad \frac{\partial}{\partial t} = -v_b\frac{\partial}{\partial \xi} \,, \qquad \frac{d}{dt} = (v - v_b)\frac{\partial}{\partial \xi} \,. \tag{4.2}$$

The current density of the beam is $j = en_b v_b$, n_p is the electron density of the unperturbed plasma, δn_p is the electron density of the perturbed plasma, and v is the velocity of the plasma electrons.

The equation of motion of a plasma electron under the action of the wake field is

$$\frac{dp}{dt} = eE, \tag{4.3}$$

where p is the momentum of the plasma electron.

Equations (4.1) and (4.3) can be expressed in the form

$$\begin{aligned} \frac{dE}{d\xi} &= 4\pi e(\delta n_p + n_b), \\ v_b\frac{dE}{d\xi} &= 4\pi e(\delta n_p + n_p)v + 4\pi e n_b v_b, \\ (v - v_b)\frac{dp}{d\xi} &= eE(\xi)\,. \end{aligned} \tag{4.4}$$

Suppose that the density of the electron beam has a simple rectangular form

$$n_b(z,t) = \begin{cases} 0 & \xi > 0 \\ n_b & -\tau < \xi < 0 \\ 0 & \xi < -\tau \end{cases}, \tag{4.5}$$

where τ is the duration of the beam pulse.

Initially the neutral plasma is unperturbed

$$E(\xi = 0) = 0,\ \delta n_p(\xi = 0) = 0,\ p(\xi = 0) = 0,\ dp(\xi = 0)/d\xi = 0. \tag{4.6}$$

Introduce the new variable α

$$d\alpha = \frac{d\xi}{v_b - v}\,. \tag{4.7}$$

As follows from (4.4) and (4.7) the plasma density perturbation is determined by the expression

$$\delta n_p = -\frac{n_p}{1 - v_b/v}\,. \tag{4.8}$$

Therefore, when the velocity of plasma electrons becomes close to the beam velocity the model breaks down. If plasma electrons perform periodic oscillations the perturbed plasma density increases drastically each time when v approaches v_b.

After substitution of the variable α, Eqs (4.4) can be reduced to a single equation for the momentum p of the plasma electrons

$$\frac{d^2p}{d\alpha^2} + 4\pi e^2 n_b v_b + 4\pi e^2(n_p - n_b)v(p) = 0\,, \tag{4.9}$$

where $v(p) = pc/\sqrt{p^2 + m^2c^2}$.

Equation (4.9) describes the motion of a material point in the effective potential field U

$$\frac{d^2p}{d\alpha^2} = -\frac{\partial U}{\partial p}, \tag{4.10}$$

where the potential field has the form

$$U(p) = 4\pi e^2 c\sqrt{p^2 + m^2c^2}\,(n_p - n_b) + 4\pi e^2 n_b v_b p \,. \tag{4.11}$$

Equations (4.10) - (4.11) describe two different cases of the motion of plasma electrons, which we discuss separately.

1. First we consider the case $n_b \leq n_p/(1+\beta_b)$, where $\beta_b = v_b/c$. In this case the potential field (4.11) has the shape of an asymmetric well (see Fig. 5). In such a well the motion of plasma electrons is finite. This means that there are oscillatory solutions for the motion of plasma electrons under the action of the electron beam pulse. These solutions move with the phase velocity equal to the beam velocity, $v_{ph} = v_b$. The oscillatory solutions exist even when the velocity of the plasma electrons is relativistic, $p \gg mc$.

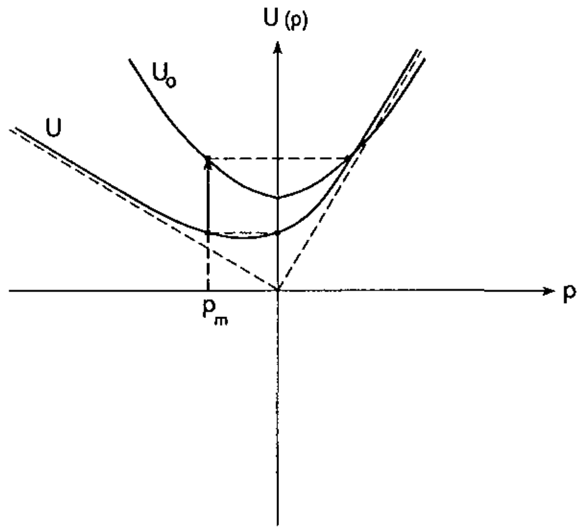

Fig. 5. The effective potential fields for $n_b \leq n_p/2$, U is in the presence of the driving bunch, U_0 is behind the bunch. Dashed lines show the motion of a particle.

The potential (4.11) is not harmonic and the oscillations correspond to the nonlinear Langmuir plasma wave. At $p \leq mc$ a transition to linear oscillations occurs.

The main features of these oscillations with $p \gg mc$ can be obtained from (4.11) and the initial conditions (4.6). The momentum of plasma electrons that corresponds to the maximum of the excited wake field, p_m, can be found from the conservation law

$$\frac{1}{2}\left(\frac{dp}{d\alpha}\right)^2 + 4\pi e^2 c\sqrt{p^2 + m^2c^2}\,(n_p - n_b) + 4\pi e^2 n_b v_b p = 4\pi mc^2(n_p - n_b)\,. \tag{4.12}$$

The derivative $dp/d\alpha$ equals to zero at the point p_m. Hence, it follows that p_m is equal to

$$p_m = -\frac{2n_b\beta_b mc(n_p - n_b)}{n_p(n_p - n_b(1 + \beta_b))}\,. \tag{4.13}$$

It is seen that p_m can be much larger then mc if n_b tends to $n_p/(1 + \beta_b)$. This means that the plasma electrons can acquire a relativistic velocity close to $- v_b$. It follows from relation (4.8) that in this case the perturbation δn_p of the plasma density tends to $- n_p/2$. At the end of the bunch, that is at the moment $\xi = -\tau$, the potential well becomes symmetric but is still anharmonic (see Fig. 5). The expression for the potential is

$$U_o(p) = 4\pi e^2 n_p c\sqrt{p^2 + m^2c^2}\,. \tag{4.14}$$

Plasma electrons perform free nonlinear oscillations in this potential with amplitude value p_m. These oscillations correspond to nonlinear relativistic Langmuir plasma waves.

It follows from (4.8) that when the velocity of the plasma electrons v approaches the beam velocity v_b, the plasma density perturbation $\delta n_p/n_p$ can become larger than unity. The longitudinal electric field of the plasma wave with such a distribution of the density is much larger than that of the linear Langmuir wave. The same is valid for the acceleration gradient, $dE_a/dz = eE$.

In the limit $p >> mc$ the amplitude of the electric field behind the beam pulse is determined by the equation that follows from (4.10) and (4.14)

$$\frac{d^2p}{d\alpha^2} + 4\pi e^2 c n_p \mathrm{sign} p = 0\,. \tag{4.15}$$

Because $eE = - dp/d\alpha$, one obtains for the maximum field strength

$$eE_{max} = \sqrt{2mc|p_m|\,\omega_p^2} \cong 2mc\omega_b\sqrt{\frac{n_b}{n_p - 2n_b}}\,, \tag{4.16}$$

where ω_b is the plasma frequency of the beam. At the point $n_p = n_b(1 + \beta_b)$ expression (4.16) diverges. The maximum plasma electron momentum can be estimated from conservation considerations which yield $p_m \leq mc\sqrt{\gamma^2 - 1}$. Thus, the amplitude of the longitudinal field is restricted to the value

$$eE_{max} < mc\omega_p\sqrt{2(\gamma^2 - 1)^{1/2}}\,. \tag{4.17}$$

In the relativistic limit $\gamma >> 1$ one obtains for the acceleration rate

$$(dE_a/dz)_{nl} \approx \gamma^{1/2}(dE_a/dz)_l\,, \tag{4.18}$$

which is much larger than the acceleration rate (2.3) for the linear LPW.

The period of the nonlinear LPW can be estimated from (4.15)

$$T_{nl} = 2\int_0^{p_m} \frac{dp}{\omega_p\sqrt{mcp_m - mcp}} = \frac{1}{\omega_p}\sqrt{\frac{|p_m|}{mc}}\,. \tag{4.19}$$

In the relativistic limit the period is restricted of

$$T \leq \gamma^{1/2}/\omega_p. \tag{4.20}$$

The minimum duration of the bunch τ that is required for the effective excitation of the NLPW should be about this period. This value exceeds the period of linear plasma oscillations by $\gamma^{1/2}$ times. The acceleration length is also $\gamma^{1/2}$ times larger than in the linear case, $L_{nl} \cong \gamma^{1/2}L$, where L is given by (2.4).

It follows from (4.18) and (4.20) that the total amount of energy that can be acquired by a trailing electron is γ times larger than the one in the BWA concept

$$E_{tot} \approx \gamma E^{1}_{tot}\,, \tag{4.21}$$

where E^{1}_{tot} is defined by (2.5).

Therefore, if the density of the driving beam n_b is close to but smaller than $n_p/2$ the nonlinear LPW can be excited. This wave is much more effective for electron acceleration than the linear LPW.

This conclusion is confirmed by preliminary 1D simulation results. The code is fully nonlinear and includes the back influence of the excited plasma wave on the electron bunch. Figure 6 shows the distribution of the density of plasma electrons and of the longitudinal electric field behind the bunch. The energy of the electron beam is 50 MeV and the beam current density is 5 kA/cm^2. This corresponds to an electron density in the bunch of 10^{12} cm^{-3}. The bunch length is 6 cm. The plasma density is $2.3\cdot10^{12}$ cm^{-3}, so that $n_b = 0.87(n_p/2)$. It is seen that the excited plasma density modulation is much larger than unity and that the amplitude of longitudinal electric field exceeds four times the amplitude of the linear Langmuir wave.

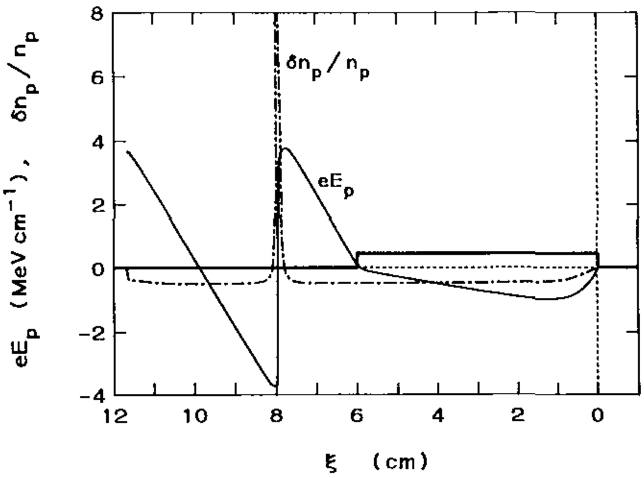

Fig. 6. The distributions of the perturbed plasma density, $\delta n_p/n_p$, and the longitudinal electric field, eE_p, behind an electron bunch. The plasma density is $2.3 \cdot 10^{12}$ cm^{-3}, the electron beam density is 10^{12} cm^{-3}, the beam-current density is 5 kA/cm^2, the bunch length is 6 cm and the energy of the beam is 50 MeV.

2. When the beam density n_b exceeds the value $n_p/2$, the potential U does not have a minimum (Fig. 7) and there is no solution for p_m. It means that no oscillatory solution for the motion of plasma electrons exists. This limit corresponds to the overturning (breakdown) of the NLPW [1]. The driving electron beam pushes all plasma electrons out of its path of propagation and leaves behind a channel populated only by plasma ions. As was mentioned in Section 3 this ion channel can be used for the production of radiation.

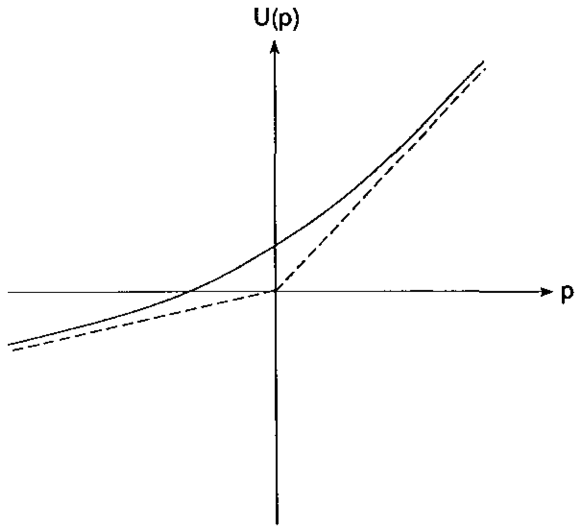

Fig. 7. The effective potential field for $n_b > n_p/2$.

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5. COMPACT SHORT WAVELENGTH COHERENT RADIATION SOURCES BASED ON DEEPLY MODULATED PLASMA STRUCTURES

The plasma FELs considered in Section 3 are based on the induced Bremsstrahlung radiation when a relativistic electron beam executes small oscillations in the transverse direction. However, any kind of radiation from relativistic electrons possessing a monochromatic spectrum and/or a sharp angular distribution can be a source of coherent radiation. This is due to the fact that the gain is proportional to the frequency and/or angular derivative of the spontaneous radiation spectrum of an electron [1]. For undulator type radiation this dependency of the gain is known as the Madey theorem [1].

In this section we will discuss FELs based on the radiation from a propagating electron beam in the absence of transverse oscillations. Transition radiation is produced when a freely moving particle passes through a step in the dielectric permittivity of the medium [2,3]. In the X-ray region the dielectric permittivity depends only on the plasma frequency, $\varepsilon' = 1 - \omega_p^2/\omega_s^2$. Therefore, situations in which the electron beam propagates either through a plasma or through any other kind of media are equivalent in this respect. The radiation is directed at a small angle θ with respect to the particle motion. This angle is smaller than the relativistic cone $1/\gamma$. The radiation frequency from a single step ranges from the optical region up to $\omega_s = \omega_p\gamma$, which corresponds to X-rays.

When the electron beam propagates through a number of periodically spaced thin foils, spontaneous transition radiation becomes resonant due to the interference of radiation emitted at different boundaries [1, 2-4]. The resonant condition depends on the period d of the structure, the foil thickness a, the radiation angle θ, and the harmonic number m:

$$\frac{\omega_s d}{4\pi v_b}\left\{(1 - \varepsilon')\frac{a}{d} + \gamma^{-2} + \theta^2\right\} = m\,. \tag{5.1}$$

It follows from (5.1) that the radiation angle has a positive value only if the radiation wavelength is confined within the limits

$$\lambda_{s,1,2} = \gamma^2\left\{\frac{m}{d} \pm \left[\left(\frac{m}{d}\right)^2 - \frac{1}{\gamma^2\lambda_p^2}\frac{a}{d}\right]^{1/2}\right\}^{-1}. \tag{5.2}$$

The condition $\lambda_s^1 = \lambda_s^2$ gives the following expression for the boundary energy for a given harmonic

$$\gamma_b = \frac{\sqrt{ad}}{m\lambda_p}\,. \tag{5.3}$$

Although transition and undulator radiation have different physical origins, both, as well as Cherenkov radiation, are similar in the case of coherent effects in a periodical structure.

To clarify the situation we consider a beam of electrons propagating through a periodic structure (see Fig. 8). This periodic structure may be either an undulator or a stack of foils or any external structure like gratings. The important feature is that in all cases the perturbation is periodic in space. The radiation at an angle θ and at a frequency ω_s will be coherent if the phase difference of photons emitted at positions shifted along z axis on the period d is proportional to 2π

$$\left\{\omega_s t - k_s z\right\}_{t=0}^{t=d/v_b} = 2\pi m, \quad m = 1,2,... \tag{5.4}$$

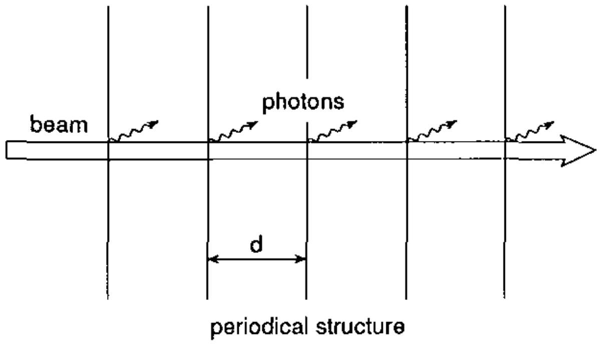

Fig. 8. Radiating electron beam moving in a periodical structure.

Therefore, this resonance condition for the first harmonic can be expressed in Cherenkov form [5]

$$\omega_s = kv_{ph} \text{ or } k = \omega_s n_{eff}/c , \tag{5.5}$$

where v_{ph} is the phase velocity of the wave, $n_{eff} = (k_s + g)/k_s$, $k = k_s + g$, $g = 2\pi/d$ is the reciprocal lattice vector. Since $n_{eff} > 1$, the phase velocity v_{ph} is less than the speed of light so that electrons can be in resonance with the emitted radiation. The radiation frequency is determined by the period of the structure and the relativistic factor, and has the well-known form for FELs

$$\omega_s = 2\gamma^2 \frac{2\pi v_b}{d} . \tag{5.6}$$

In both cases the radiation is connected with the transformation of the momentum to quasi-momentum as a result of the interaction of an electron (undulator) or a photon (coherent transition radiation-CTR) with the periodical structure.

It can be shown that the gain per pass is given by the well-known formula of FEL physics

$$G \sim p^2 d^{1/2} \lambda_s^{3/2} N^3 j_b , \tag{5.7}$$

when $\Delta\gamma/\gamma < 1/2N$, where N is the number of periods, j_b is the beam current density, and by

$$G \sim p^2 d^{1/2} \lambda_s^{3/2} \left(\frac{\gamma}{\Delta\gamma}\right)^2 N j_b , \tag{5.8}$$

when $\Delta\gamma/\gamma > 1/2N$. The pump parameter p is determined by the relation [1,5]

$$p = \Delta\theta_d / \Delta\theta_\gamma , \tag{5.9}$$

where $\Delta\theta_\gamma = \gamma^{-1}$ is the radiation angle of the relativistic electron. For undulators, $\Delta\theta_d$ is the deviation angle of the electron velocity from its initial direction at the end of the half period, $\Delta\theta_d = edB_u/2\pi\gamma mc^2$, where B_u is the amplitude of the undulator field. For coherent transition radiation (CTR), $\Delta\theta_d$ is the deviation angle of the wave vector of the corresponding Furie component of the electron field (the electron itself moves straightly), $\Delta\theta_d\,\gamma = \delta n_p/n_p$, $\delta n_p/n_p$ is the degree of modulation of the electron density of the periodic medium. The probability of the transformation of the photon momentum into quasi-momentum is maximal if the pump parameter is near unity, $p \approx 1$. Hence, the most effective production of CTR requires plasma structures with deeply modulated electron density, $\delta n_p/n_p \sim 1$.

The present state of the art of undulator construction provides undulators with $p \approx 1$ and periods of cm scale. The latter fact implies that high energy electron beams are required for short wavelength (VUV or X-ray) production. Modulated plasma structures can have much shorter periods and, therefore, short wavelength radiation can be produced with a moderate energy electron beam. In addition, the total length of such a radiator can also be very short. The gain is significantly reduced in the short wavelength range. According to (5.7) and (5.8) this reduction can be compensated by increasing the number of periods.

In conclusion, a short wavelength generator with a sufficient gain requires a deeply modulated plasma with a short periodicity length and a large number of periods. In what follows we will examine these requirements for a number of plasma structures.

5.1. Nonlinear LPW. According to Section 4 a powerful electron beam can produce highly modulated plasma densities. The electron beam which propagates through the plasma excites a nonlinear travelling Langmuir plasma wave if the beam density is close to but just below $n_p/(1+\beta)$, where $\beta = v_b/c$. The average degree of modulation of this wave is $\delta n_p/n_p \sim 1$ and the period d is equal to $d = \lambda_p\sqrt{\gamma}$, where γ is the relativistic factor of the driving beam.

When the beam is weakly relativistic and the beam current density j_b is of the order of several hundreds kA/cm^2, structures with $d \sim 1$ mm will be created. Another beam with a higher energy propagating in the opposite direction will produce the radiation (Fig. 9).

A well-defined periodic modulation of the electron density can only be maintained as long as ions remain immobile. The energy introduced into the plasma by the driving beam pumps also modulational and parametric instabilities that spoil the periodicity. Ion motion sets the following limit to the number of periods of the density modulation

$$N \leq \sqrt{\omega_{pe}^2/\omega_{pi}^2} = \sqrt{\frac{m_i}{m_e}}, \tag{5.10}$$

where m_e , m_i are the electron and ion masses, respectively.

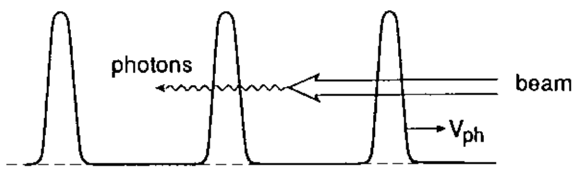

Fig. 9. Radiator based on nonlinear LPW or Langmuir solitons ($v_{ph} \approx 0$).

Up to now we have considered the nonstationary situation. In the next section we will discuss stationary processes.

5.2. Langmuir solitons. During plasma heating by RF radiation or by electron beams, the heating energy is initially transferred to Langmuir oscillations. Langmuir waves are subjected to induced scattering by electrons and ions, and their energy is transmitted to lower wavenumbers. With the increase of the energy of Langmuir waves, local regions of high amplitude electric fields are formed and the electrons are pushed out from these regions. As a result a standing (or slowly moving) periodic structure of solitons is created with a deeply modulated density [6,7].

The behaviour of the modulational instability is defined by the relation between the spreading of a linear wavepacket and the compression due to nonlinearity. For Langmuir waves the dispersion relation is

$$\omega = \omega_p + 3/2\,(kr_D)^2\omega_p - W\omega_p/2n_pT\,, \tag{5.11}$$

where $r_D = (T/4\pi n_p e^2)^{1/2}$ is the electron Debye radius and W is the density of the electric energy. The wavepacket is compressed when $W > 3/2\,(\chi r_D)^2$, where $\chi \cong \pi/\Delta$ is the range of wavenumbers in the packet, Δ is the longitudinal dimension of the soliton. The field amplitude E_0 and the width Δ of the soliton are related by $E_0\Delta$ = const, so that the energy of a soliton $W \sim E_0^2\Delta$, increases as $W \sim 1/\Delta$ if Δ decreases. Since $(\chi r_D)^2$ is proportional to Δ^{-2}, the collapse will stop. The density modulation $\delta n_p/n_p = W/n_pT$ that is produced is determined by the ratio of the energy density in the soliton and the kinetic energy density of the plasma.

The collapsing cavities are developed during a time period of the order of some hundreds or even tens of Langmuir oscillations. Periodic bunches of Langmuir solitons produced by the monoenergetic electron beam were observed experimentally in a magnetized collissionless plasma of low density ($\sim 10^9$ cm^{-3}) [8,9]. The number of solitons was about 25 and very strong solitons characterized by $\delta n_p/n_p \approx 0.3$ were observed. For the conditions of the experiment the period was d = 2-3 cm, the size of a soliton was Δ = 0.3-0.4 cm. The value of the plasma density was chosen from the point of resolution of the applied apparatus. At higher densities smaller periods can be reached. It is difficult to observe solitons experimentally in a rather dense, fully ionized and unmagnetized plasma, but there is no principal restriction for the density. The appearance of the collapsing cavities is well confirmed by numerical experiments [6] which show that the parametric plasma instability at the nonlinear stage can cause the creation of numerous periodical cavities [7].

5.3. Stack of foils (Fig. 10). A stack of foils can be considered as an artificially prepared structure with a high degree of the modulation of the plasma density. In this case the degree of modulation $(n_{p1} - n_{p2})/n_{p1} \sim 1$, where n_{p1} is the density of electrons of the foil substance and n_{p2} is the electron density of the medium between foils. The foil thickness can be as low as 1 μm and the period can be as small as 10 μm. The foil thickness should exceed the coherence length in the material, $a > l_c$, and the distance between the foils should be larger than the coherence length in vacuum l_v, $d-a > l_v$, where l_c and l_v are given by [3,10]

$$l_c = \frac{\lambda_s}{\gamma^{-2} + \theta^2 + \lambda_s^2/\lambda_p^2}, \qquad l_v = \frac{\lambda_s}{\gamma^{-2} + \theta^2}. \tag{5.12}$$

In the X-ray region $\theta^2 << \gamma^{-2}$ and $\lambda_s \sim \lambda_p\gamma$, so that $l_c, l_v \sim \lambda_p\gamma$. In the case of a metal foil the wavelength λ_p is typically of the order 50-80 nm, so that $l_c, l_v \sim 1$ μm.

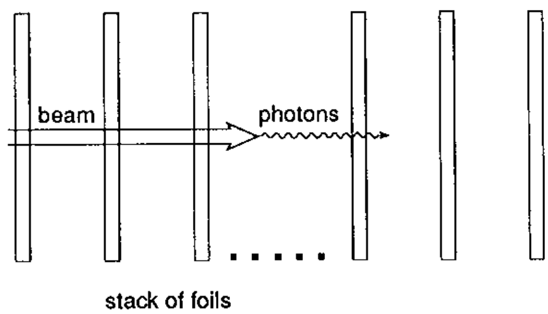

Fig. 10. Radiating electron beam moving through the stack of foils.

In the experiment reported in Ref. [4] the observed spectral intensity (photons/sr/eV/electron) in soft X-ray (the photon energy ~ 1 keV) was of the same order of magnitude as the synchrotron emission at 4.5 GeV. The gain is very low in this case due to the limited number of foils. With a larger number of foils the total length of the medium L = Na increases and multiple scattering of the particle will become important. The mean square angle of the multiple scattering is described by the relation

$$< \theta_s^2 > = \frac{\gamma_s^2}{\gamma^2} \frac{L}{L_{rad}}, \tag{5.13}$$

where L_{rad} is the radiation length [2], which for light elements is about several tens of cm, γ_s is the characteristic factor, $\gamma_s \approx 30$. According to (5.7), the gain is $G \sim N^3$, as long as the condition $< \theta_s^2 > \gamma^2 < N^{-1}$ is fulfilled. This condition is rather severe and limits the above scaling of the gain to $N \leq 50$. The same condition would require a very low electron beam emittance for operation with a usual undulator in X-rays. For a large number of periods the gain is given by (5.8) and is proportional to the number of periods. Since the angle of multiple scattering for the length equals to the coherence length, l_c, is small, multiple scattering from a single foil is negligible. The gain is also proportional to the derivative of the electron distribution function. Hence, the equation describing the evolution of the distribution function should include a collision term. The latter question requires special investigation.

In principle, the problem of multiple scattering can be avoided by making a small hole in the foil. The size of the hole should not exceed $\lambda_s \gamma$. The beam size must have comparable dimensions.

Another problem that should be studied is the photo-absorption. The photo-absorption reduces the gain and this reduction depends on the mass absorption coefficient. The mass absorption coefficient is determined by the imaginary part of the dielectric permittivity, ε'', which is lower for light elements. In materials like Be, the photo-absorption length can be as long as 1 cm for 10 keV photons. This means that the number of periods can be as large as 10^3, before photo-absorption becomes dominant.

5.4. Artificially modulated plasma. The photo-absorption problem can be avoided by the conversion of the foils or a grating into the plasma state by a powerful electron beam or by a laser radiation (Fig. 11). Transition radiation does not depend on small-scale variations of the angle at which an electron crosses the boundary between two media and details of the boundary configuration are not important if the size of the diffused region is smaller than the coherence length (5.12). The coherence length of the generated X-ray radiation is always sufficiently long, so that the requirements on the boundary are not severe. The requirement on the regularity of the structure $\Delta d/d << N^{-1}$ is identical to the one for a standard undulator.

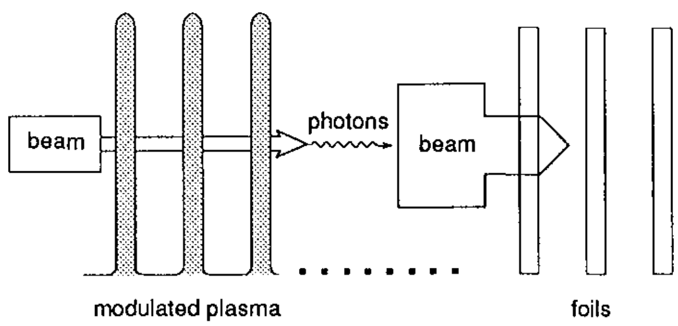

Fig. 11. Radiator based on artificially prepared deep plasma density modulation.

5.5. Z-pinch with artificially prepared necks. Z-pinch configurations can be created in different ways. One of these is by the explosion of thin wires in diodes of high current accelerators [11]. The produced plasma column is unstable with respect to the sausage instability. As a result the column narrows and necks-in symmetrically (Fig. 12). The result of this process is a strongly modulated, dense, periodic structure. The periodicity scale is set by the wavelength of the perturbation. The evolution of the necks goes through two stages: at first a long-wavelength perturbation develops, subsequently a small neck appears on this background. Typical z-pinch parameters are given in Ref. [12]. The electron density outside the neck is 10^{21} cm^{-3} and inside the neck 10^{23} cm^{-3}, the pinch length is 0.1-1 cm, and the period is 0.01-0.1 cm. The average magnetic field is 10 MG and can reach the value 100 MG inside the neck. The lifetime is about 100 ns or less. The magnetic field is created by a current pulse with the typical magnitude I = 1 MA.

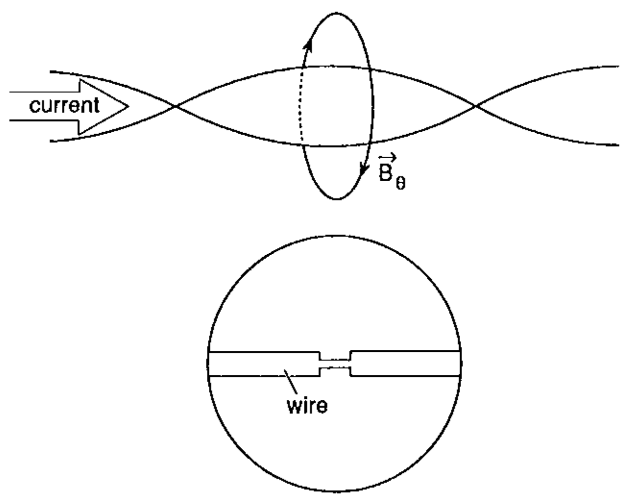

Fig. 12. Z-pinch necks and the profile of the wire.

The position of the necks and the distance between them are arbitrary parameters. However, there are indications from Kurchatov z-pinch experiments [12] that position of the necks can be fixed by preparing the profile in advance (Fig. 12) by making periodic scratches on the wire. For FEL applications more periods are required, and thus the study of z-pinches with longer wires and with higher density of scratches is necessary.

Uniform z-discharges have already been used for the demonstration of highly efficient transport of ion beams [13]. Z-pinch is also a promising tool for focusing high energy particles in an accelerator [14]. The very strong magnetic field with high longitudinal gradients could provide a substantial transverse compression of the beam up to the radius of the neck. According to (5.7) and (5.8), this compression will enhance the gain.

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6. CONCLUSIONS

A plasma is a medium in which small-scale structures accompanied by high electric fields and large field gradients can be excited. These properties make plasmas attractive for the production of compact accelerators. In addition, the regular patterns of deeply modulated density structures that can be created in plasmas offer possibilities for compact undulators for FELs in short wavelength regimes.

In this report a brief review is presented of the various concepts in which plasma phenomena are or can be used for these purposes.

The emphasis is on the physical aspects and on the conceptual possibilities and limitations on acceleration rates and on the period and degree of modulation of density structures.

The new concept of the beam excited nonlinear plasma wave presented in Section 4, is of great interest because it promises ultra high acceleration gradients and final energies. In the wake of the beam pulse, a deeply modulated periodic density structure with a short periodicity length appears. In analogy with standard magnetic undulators, this structure could be used to excite coherent, short wavelength radiation. This concept is largely unexplored and deserves a thorough study of its potentialities. As a first step, the efficiency of excitation of a LPW, the degree of modulation and its periodicity, and the requirements on the electron beam should be investigated analytically and numerically.

A FEL system for VUV or X-ray radiation based on coherent transition radiation from an electron beam passing through an externally modulated plasma structure is a promising alternative to the long undulators installed in storage rings. An example of such a structure is the stack of foils discussed in Section 5. The main advantage of such a system is its short periodicity length so that the length of the radiator can be one to two orders of magnitude shorter than a standard magnetic undulator. Moreover, the required beam energy can be an order of magnitude lower than that of a storage ring.