Resistive plasmas

Overview

- **Introduction:** magnetic explosions;
- **Stability of force-free magnetic fields:** Lundquist field, a trap;
- **Resistive instabilities:** tearing mode analysis;
- **Spectral theory of resistive modes.**

Magnetic explosions

- Recall MHD8-24 (observation):
  TRACE movie of erupting filament
  (T171-10050616-19-21-filerup.mpeg)

- Recall MHD8-41 (simulation):
  CME by flux cancellation
  (Mikic-flx2d.anim.qt)

Force-free field in cylinder

- **Stability of Lundquist field**

$$\mathbf{j} = \alpha \mathbf{B} \;\Rightarrow\; B_z = B_0 J_0(\alpha r)\,, \; B_\theta = B_0 J_1(\alpha r)\,.$$

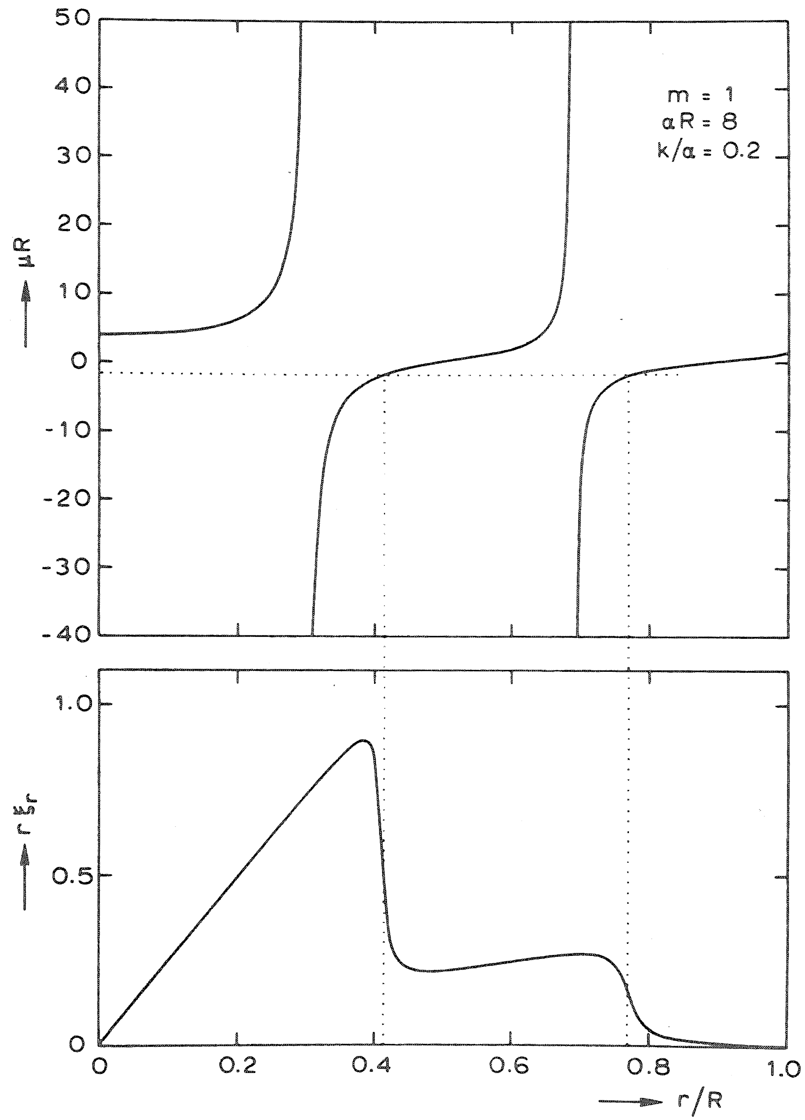
Energy principle analysis

[Voslamber & Callebaut, Phys. Rev. **128**, 2016 (1962)].

Normal mode analysis

[Goedbloed & Hagebeuk, Phys. Fl. **15**, 1090 (1972)],

Field pitch $\mu \equiv \dfrac{B_\theta}{rB_z}$, and eigenfunction $\Rightarrow$

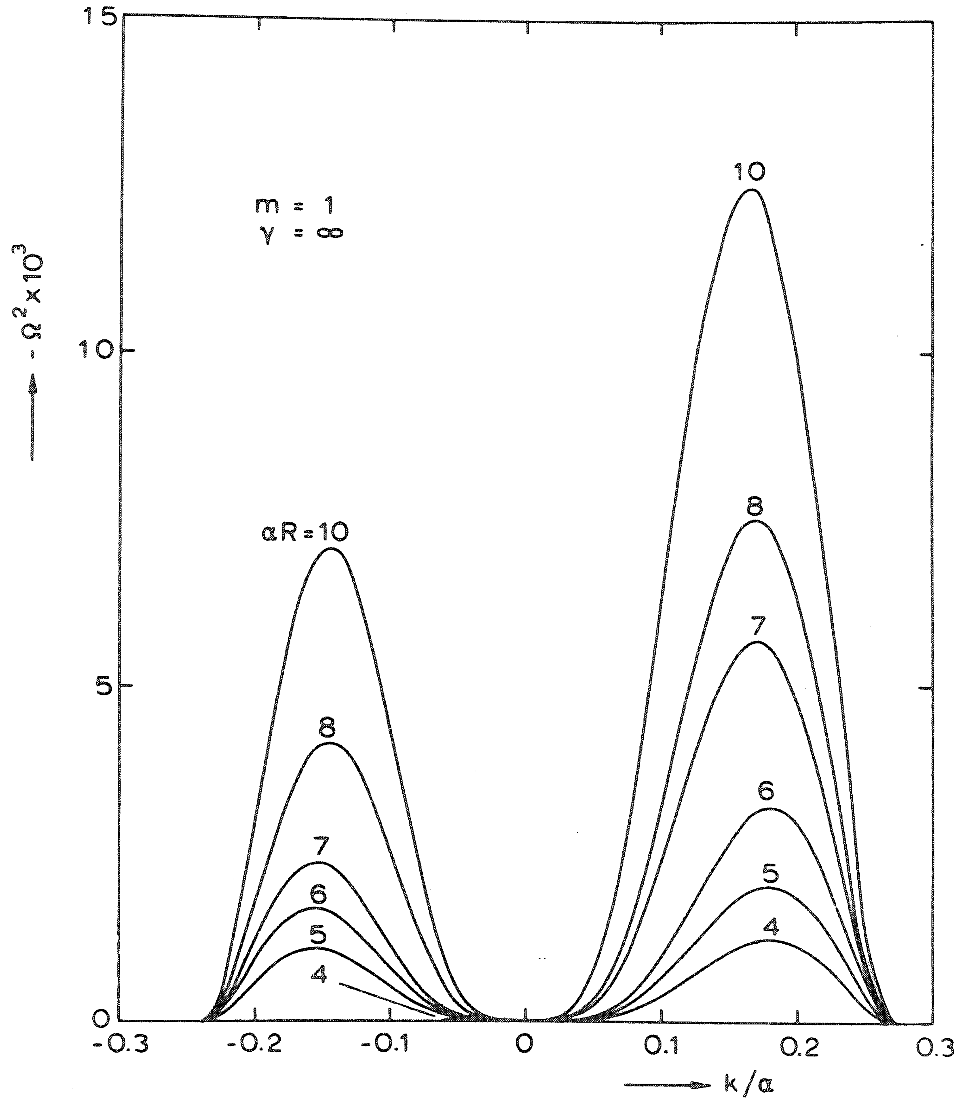


Force-free field in cylinder (cont'd)

- **Stability of Lundquist field**

  [Goedbloed & Hagebeuk, PF **15**, 1090 (1972

  Growth rates ⇒



Force-free field in plane geometry

- Plasma confined between two perfectly conducting plates at $x = x_1$ and $x = x_2$. No gravity, no pressure, only current and magnetic field.

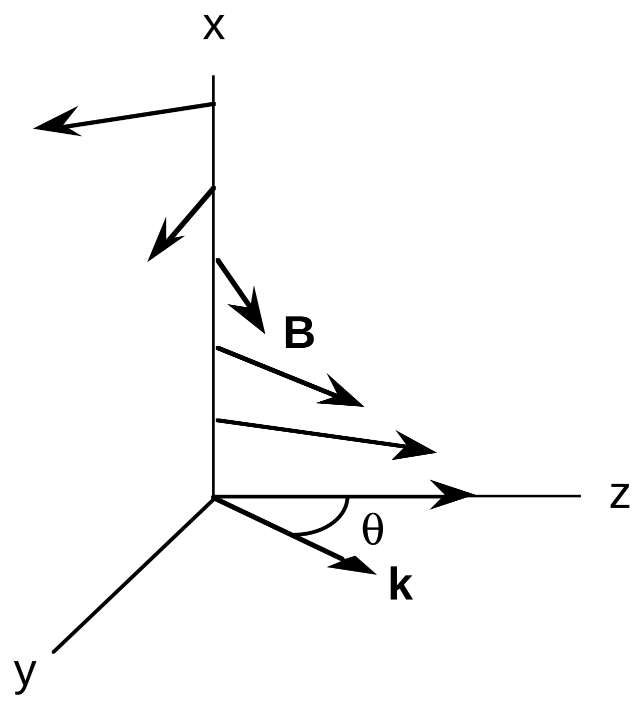
- Current such that the magnetic field has constant magnitude but its direction varies.
  $\Rightarrow$ **Force-free magnetic field with constant α** (assumption):

$$\mathbf{j} = \nabla \times \mathbf{B} = \alpha \mathbf{B} \quad \Rightarrow \quad j_y = -B_z' = \alpha B_y \,, \quad j_z = B_y' = \alpha B_z \,. \tag{1}$$

- These equations can easily be integrated:

$$B_y = B_0 \sin \alpha x \,, \quad B_z = \cos \alpha x \,. \tag{2}$$

  The parameter αa measures the total current through the plasma.



- **Is this configuration stable?**

  (Is there a limit to the value of αa?)

Stability: first method (exploiting Q)

- Exploit energy principle: [following Schmidt, Physics of High Temperature Plasmas (1979)]

$$W = \tfrac{1}{2} \int \big(\mathbf{Q}^2 + \alpha \mathbf{B} \cdot \boldsymbol{\xi}^* \times \mathbf{Q} \big) \, dx \,. \tag{3}$$

- Introduce vector potential $\mathbf{A}$,

$$\mathbf{Q} = \nabla \times \mathbf{A} \,, \qquad \mathbf{A} \equiv \boldsymbol{\xi} \times \mathbf{B} \,, \tag{4}$$

$$\Rightarrow \quad W = \tfrac{1}{2} \int \big[(\nabla \times \mathbf{A})^2 - \alpha \mathbf{A}^* \cdot \nabla \times \mathbf{A} \big] \, dx \,. \tag{5}$$

- Minimize W subject to some convenient normalization. Choose the helicity: [Woltjer, Proc. Nat. Acad. Sci. USA **44**, 489 (1958); Taylor, PRL **33**, 1139 (1974)]

$$K \equiv \tfrac{1}{2} \int \mathbf{A}^* \cdot \nabla \times \mathbf{A} \, dx = \mathrm{const} \,. \tag{6}$$

- This problem is equivalent to minimization of the quadratic form

$$\widetilde{W} \equiv W + \lambda K = \tfrac{1}{2} \int \big[(\nabla \times \mathbf{A})^2 - (\alpha - \lambda) \mathbf{A}^* \cdot \nabla \times \mathbf{A} \big] \, dx \,, \tag{7}$$

where λ is a Lagrange multiplier that is to be determined together with $\mathbf{A}$.

First method (cont'd)

- Integrate by parts:

$$\widetilde{W} = \tfrac{1}{2}\Big[\mathbf{A}^* \times (\nabla \times \mathbf{A}) \cdot \mathbf{n}\Big]_{x_1}^{x_2} + \tfrac{1}{2}\int \mathbf{A}^* \cdot [\nabla \times \nabla \times \mathbf{A} - (\alpha - \lambda)\nabla \times \mathbf{A}]\, dx\,. \quad (8)$$

Boundary term vanishes because of BCs $\mathbf{B} \cdot \mathbf{n} = 0$ and $\boldsymbol{\xi}^* \cdot \mathbf{n} = 0$.

- For arbitrary $\mathbf{A}^*$, $\widetilde{W}$ is minimized by solutions of the Euler–Lagrange equation

$$\nabla \times \nabla \times \mathbf{A} - (\alpha - \lambda)\nabla \times \mathbf{A} = 0\,. \quad (9)$$

which is a force-free field equation for the perturbations:

$$\nabla \times \mathbf{Q} = \tilde{\alpha}\,\mathbf{Q}\,, \qquad \tilde{\alpha} \equiv \alpha - \lambda\,. \quad (10)$$

- Eigenfunctions and eigenvalue $\tilde{\alpha}$ (and hence λ) determined by BCs $\mathbf{n} \cdot \mathbf{Q}|_{x_{1,2}} = 0$. Inserting such an eigenfunction into (8) gives $\widetilde{W} = 0$, so that

$$W = \widetilde{W} - \lambda K = (\tilde{\alpha} - \alpha)\,\tfrac{1}{2}\int \mathbf{A}^* \cdot \nabla \times \mathbf{A}\, dx = \frac{\tilde{\alpha} - \alpha}{\tilde{\alpha}}\,\tfrac{1}{2}\int \mathbf{A}^* \cdot \nabla \times \nabla \times \mathbf{A}\, dx$$
$$= \frac{\tilde{\alpha} - \alpha}{\tilde{\alpha}}\,\tfrac{1}{2}\int (\nabla \times \mathbf{A})^2\, dx = \frac{\tilde{\alpha} - \alpha}{\tilde{\alpha}}\,\tfrac{1}{2}\int \mathbf{Q}^2\, dx\,. \quad (11)$$

First method (cont'd)

- Hence, $W < 0$ (system unstable) if equation (10) has eigenvalue $\tilde{\alpha}$ in the range

$$0 < \tilde{\alpha} < \alpha \,. \tag{12}$$

It remains to determine that eigenvalue.

- We have to study the Euler equation (10) in detail to find out whether this condition can be satisfied. Reduction yields an ODE for the normal component of $\mathbf{Q}$:

$$Q'' + (\tilde{\alpha}^2 - k^2)Q = 0 \,. \tag{13}$$

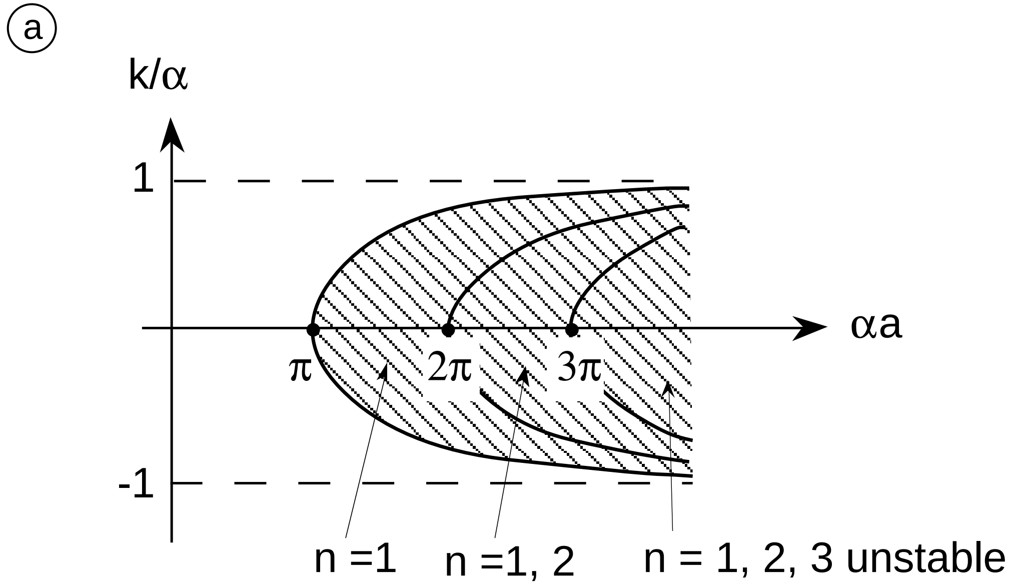
- The solution of Eq. (13) which vanishes for $x = x_1$ and $x = x_2$ reads:

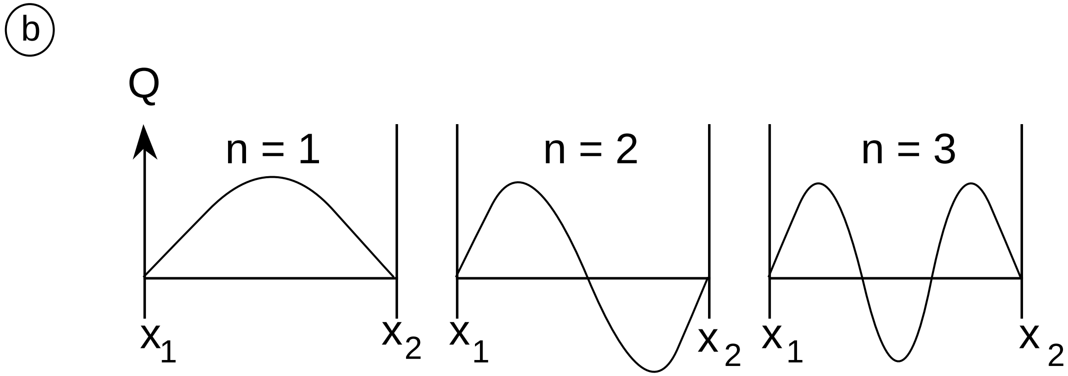
$$Q = C \sin \sqrt{\tilde{\alpha}^2 - k^2}x \,, \quad \text{where} \ \sqrt{\tilde{\alpha}^2 - k^2} = n\pi/a \,, \quad a \equiv x_2 - x_1 \,. \tag{14}$$

Hence, the instability criterion (12) is fulfilled for

$$\tilde{\alpha}^2 = k^2 + \frac{n^2\pi^2}{a^2} < \alpha^2 \,, \quad \text{or} \quad (k/\alpha)^2 + (n\pi/\alpha a)^2 < 1 \,. \tag{15}$$

This gives an unstable region in the $k/\alpha - \alpha a$ plane as sketched on following page.





- Moving to the right in shaded area subsequently $n = 1\,, n = 2\,, \ldots$ become unstable. The marginal modes ($\tilde{\alpha} = \alpha$) are labeled by the number of nodes of Q.

- For long wavelengths ($k = 0$), if αa increases with πa (magnetic field changes direction by $180°$), a new mode with one more node becomes unstable.

- Appears to be reasonable result:

  a long wavelength instability driven by the current, which has to surpass critical value given by $\alpha a = \pi$.

Stability: second method (exploiting ξ)

- Double-check the result obtained by rederiving it from formulation in terms of the displacement $\boldsymbol{\xi}$ rather than the magnetic field perturbation $\mathbf{Q}$.
- Exploit *field line projection* $\boldsymbol{\xi} = \xi \mathbf{e}_x - \mathrm{i}\eta \mathbf{e}_\perp - \mathrm{i}\zeta \mathbf{e}_\parallel$, and express the components of $\mathbf{Q} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$ in terms of ξ, η and ζ (noting that the latter does not appear):

$$
\begin{aligned}
Q_x &= \mathrm{i}F\xi \,, \\
Q_y &= -(B_y\xi)' + k_z B\eta = -B_y\xi' - \alpha B_z\xi + k_z B\eta \,, \\
Q_z &= -(B_z\xi)' - k_y B\eta = -B_z\xi' + \alpha B_y\xi - k_y B\eta \,,
\end{aligned}
\qquad (16)
$$

- It is now straightforward to express the potential energy (3) in terms of ξ and η:

$$
\begin{aligned}
W &= \tfrac{1}{2}\int_{x_1}^{x_2} [|Q_x|^2 + |Q_y|^2 + |Q_z|^2 - \alpha\xi_x^*(B_yQ_z - B_zQ_y) - \alpha(B_z\xi_y^* - B_y\xi_z^*)Q_x]\, dx \\
&= \tfrac{1}{2}\int_{x_1}^{x_2} [F^2\xi^2 + (\alpha B\xi - F\eta)^2 + (B\xi' + G\eta)^2 - \alpha^2B^2\xi^2 + 2\alpha BF\xi\eta]\, dx \\
&= \tfrac{1}{2}\int_{x_1}^{x_2} [F^2(\xi^2 + \eta^2) + (B\xi' + G\eta)^2]\, dx > 0 \,.
\end{aligned}
\qquad (17)
$$

$\Rightarrow$ **The slab is trivially stable!** [Goedbloed & Dagazian, Phys. Rev **A4**, 1554 (1971)]

Second method (cont'd)

- We may obtain the minimizing perturbations by rearranging terms:

$$W = \tfrac{1}{2}\int_{x_1}^{x_2} [F^2(\xi'^2/k^2 + \xi^2) + (kB\eta + G\xi'/k)^2]\, dx \,,$$

  so that W is minimized for perturbations that satisfy

$$kB\eta + G\xi'/k = 0\,, \quad \text{and} \quad (F^2\xi')' - k^2F^2\xi = 0\,. \qquad (18)$$

- One easily checks that the latter equation corresponds to Eq. (13) for $\tilde{\alpha} = \alpha$ when $Q = F\xi$ is substituted: the minimising equations are equivalent.

  $\Rightarrow$ **There is no mistake in the algebra!**

What went wrong?

- To see what went wrong, plot the eigenfunctions ξ corresponding to the eigenfunctions Q shown on R-9. Writing $F = kB\cos(\alpha x - \theta)$, we find:

$$\xi = \frac{Q}{F} = \frac{1}{kB}\,\frac{\sin(n\pi x/a)}{\cos(\alpha x - \theta)}\,. \qquad (19)$$

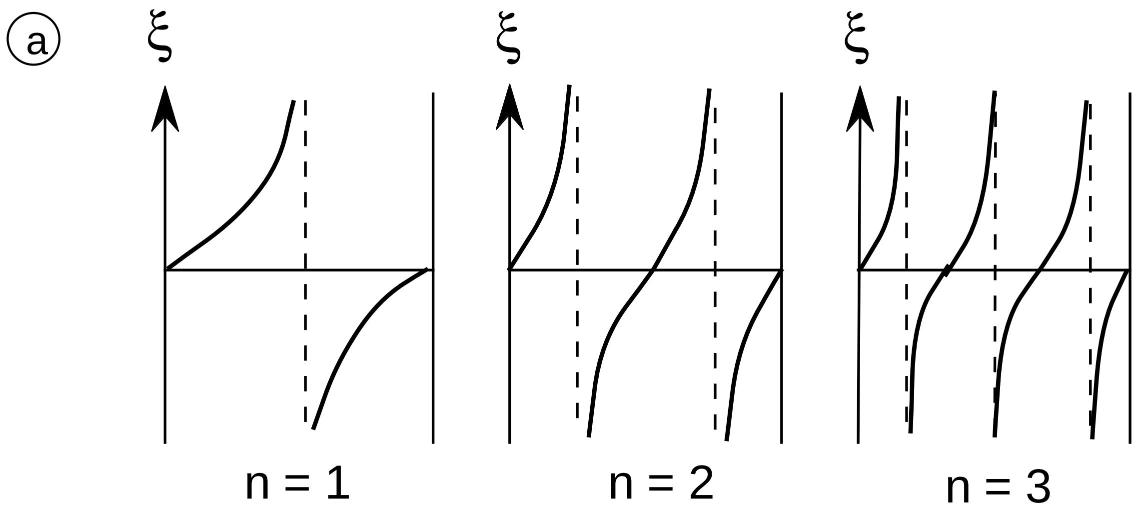
  Hence, if a solution Q exists such that W as given in Eq. (11) is negative, $\alpha a > \pi$ and ξ develops a *singularity* (see following page). For every zero that is added in Q, at least one zero is added to the function F because F oscillates faster than or at least as fast as Q.

- These singularities are such that the norm

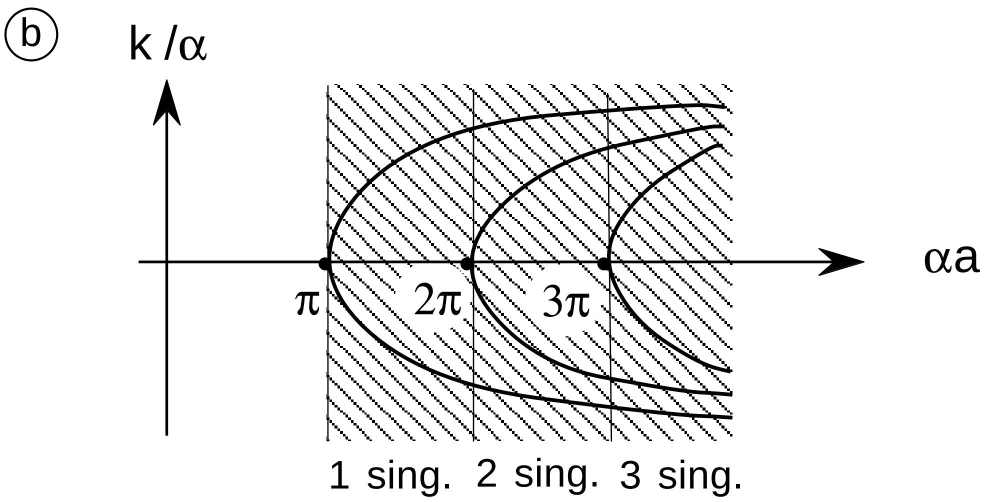
$$\|\boldsymbol{\xi}\|^2 = \int (\xi^2 + \eta^2 + \zeta^2)\,\rho\,dx = \int [\,\xi^2 + G^2\xi'^2/(k^4B^2)]\,\rho\,dx \to \infty\,,$$

  where η from Eq. (18) and $\zeta = 0$ have been substituted.

- Hence, **the trial functions Q used in deriving the stability criterion (15) do not correspond to permissible displacements ξ.**



- Marginal modes in terms of ξ.



- Singularities of ξ occur for $\alpha a > \pi$ in the shaded area:

  there is always a singularity in the 'unstable' regions of R-9.

- **Is that all: no use for this nice stability diagram?**

Basic equations

- We now present the **resistive normal mode analysis** of the plane slab. Starting point is the nonlinear resistive MHD equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \,, \tag{20}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \rho \mathbf{g} + \mathbf{j} \times \mathbf{B} \,, \qquad \mathbf{j} = \nabla \times \mathbf{B} \,, \tag{21}$$

$$\frac{\partial p}{\partial t} = -\mathbf{v} \cdot \nabla p - \gamma p \nabla \cdot \mathbf{v} + (\gamma - 1)\, \eta |\mathbf{j}|^2 \,, \tag{22}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \mathbf{j}) \,. \tag{23}$$

  Resistivity causes Ohmic dissipation term in the pressure equation and **resistive diffusion in the flux equation.** The latter completely changes the stability analysis.

- We linearise the equations for perturbations about static equilibrium. Strictly, this assumption is not justified since resistivity causes magnetic field to decay. However, *the magnetic Reynolds number $R_m \equiv \mu_0 l_0 v_A/\eta$ is usually very large* so that this is a very slow process: $\tau \sim R_m \cdot \tau_A$, where τ_A is the characteristic Alfvén time for ideal MHD phenomena. The resistive modes grow on the much faster time scale $\sim (R_m)^\nu$, where $0 < \nu < 1$, so that the equilibrium may be considered static.

- Linearize:
$$f(\mathbf{r}, t) = f_0(x) + f_1(x)\mathrm{e}^{\mathrm{i}(k_y y + k_z z - \omega t)}\,, \tag{24}$$
with equilibrium variables ρ_0, p_0, and $\mathbf{B}_0$, where we will suppress the subscript 0, and perturbations variables $\delta \equiv \rho_1$, $\mathbf{v} \equiv \mathbf{v}_1$, $\pi \equiv p_1$, and $\mathbf{Q} \equiv \mathbf{B}_1$.

- Assuming constant resistivity η, the linearised evolution equations read:
$$\frac{\partial \delta}{\partial t} = -\nabla \cdot (\rho \mathbf{v})\,, \tag{25}$$
$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla \pi + \delta \mathbf{g} - \mathbf{B} \times (\nabla \times \mathbf{Q}) + (\nabla \times \mathbf{B}) \times \mathbf{Q}\,, \tag{26}$$
$$\frac{\partial \pi}{\partial t} = -\mathbf{v} \cdot \nabla p - \gamma p \nabla \cdot \mathbf{v} + 2(\gamma - 1)\eta\, \nabla \times \mathbf{B} \cdot \nabla \times \mathbf{Q}\,, \tag{27}$$
$$\frac{\partial \mathbf{Q}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{Q}\,. \tag{28}$$
The resistive terms spoil possibility of integrating the equations for δ, π, and $\mathbf{Q}$ to get expressions in terms of $\boldsymbol{\xi}$ alone (as in ideal MHD). We can still exploit $\boldsymbol{\xi}$, but it will not be possible to eliminate $\mathbf{Q}$. Thus, the new feature of resistive MHD is the distinction between fluid flow, described by $\boldsymbol{\xi}$, and magnetic field evolution, described by $\mathbf{Q}$: **Magnetic field and fluid do not necessarily move together anymore**.

- *Project onto the direction of inhomogeneity (x) and $\mathbf{k}_0 = (0, k_y, k_z)$:*

$$\begin{aligned}
u &\equiv v_x && \textit{(normal velocity)}\,, \\
v &\equiv (\nabla \times v)_x = -\mathrm{i}(k_z v_y - k_y v_z) && \textit{(normal vorticity)}\,, \\
w &\equiv \nabla \cdot \mathbf{v} - {v_x}' = \mathrm{i}(k_y v_y + k_z v_z) && \textit{(horizontal compressibility)}\,, \\
Q &\equiv -\mathrm{i}Q_x && \textit{(normal magnetic field)}\,, \\
R &\equiv \mathrm{i}(\nabla \times Q)_x = \mathrm{i}\, j_{1x} = k_z Q_y - k_y Q_z && \textit{(normal current)}\,.
\end{aligned} \tag{29}$$

- The eigenvalue problem becomes

$$\begin{aligned}
-\mathrm{i}\omega\,\delta &= -(\rho u)' - \rho w\,, \\
-\mathrm{i}\rho\omega\,u &= -\pi' - g\delta + k_0^{-2}(FQ' + GR)' - FQ\,, \\
-\mathrm{i}\rho\omega\,v &= -G'Q + FR\,, \\
-\mathrm{i}\rho\omega\,w &= k_0^2\pi - F'Q - GR\,, \\
-\mathrm{i}\omega\,\pi &= -\,p'u - \gamma p(u' + w) - 2(\gamma - 1)\eta\, k_0^{-2}\left[\,F'(Q'' - k_0^2 Q) + G'R'\,\right], \\
-\mathrm{i}\omega\,Q &= Fu + \eta(Q'' - k^2 Q)\,, \\
-\mathrm{i}\omega\,R &= (Gu)' - Fv + Gw + \eta(R'' - k^2 R)\,,
\end{aligned} \tag{30}$$

where F and G are the projections of $\mathbf{k}_0$ onto the magnetic field.

- Assume **incompressibility:** take limits $\gamma \to \infty$ and $\nabla \cdot \mathbf{v} \to 0$ such that product $\gamma p \nabla \cdot \mathbf{v}$, and hence π, remains finite but undetermined. Consequently, equation for π should be dropped *(Ohmic dissipation term disappears from the problem)* and replaced by the constraint of incompressibility, $\nabla \cdot \mathbf{v} = 0$. Hence, $w = -u'$, so that Eq. (30)(d) for w can then be used to determine π. Variables δ, v, and w may be expressed in $\xi \equiv u/(-\mathrm{i}\omega)$, Q, and R so that we obtain a 6th order sytem:

$$\begin{aligned} &\eta\,[\,(\rho\omega^2\xi')' - k^2(\rho\omega^2 + \rho' g)\,\xi + F''Q\,] + \mathrm{i}\omega F\,(Q - F\xi) = 0\,, \\ &\eta\,(Q'' - k^2 Q) + \mathrm{i}\omega\,(Q - F\xi) = 0\,, \\ &\eta\,(R'' - k^2 R) + \mathrm{i}\omega\,(R - G'\xi) - \frac{\mathrm{i}F}{\rho\omega}(FR - G'Q) = 0\,. \end{aligned} \qquad (31)$$

  Last equation does not couple so that we get a *4th order system for ξ and Q alone.*

- *Ideal MHD limit $\eta \to 0$ is tricky:* Expand Eq. (31)(b) to first order,

$$Q = F\xi + \frac{\mathrm{i}\eta}{\omega}(Q'' - k^2 Q) \approx F\xi + \frac{\mathrm{i}\eta}{\omega}[\,(F\xi)'' - k^2 F\xi\,]\,, \qquad (32)$$

  and then insert result in Eq. (31)(a):

$$[(\rho\omega^2 - F^2)\,\xi']' - k^2(\rho\omega^2 - F^2 + \rho' g)\xi = 0\,, \qquad Q = F\xi\,. \qquad (33)$$

  This agrees with ideal MHD, where field and fluid move together again.

Tearing analysis

- *Tearing modes* result in breaking and rejoining of the magnetic field lines, and are exponentially unstable so that a real and positive eigenvalue may be defined:

$$\lambda \equiv -\mathrm{i}\omega > 0\,; \tag{34}$$

We assume $\rho = \mathrm{const} \;\Rightarrow\; \rho' g = 0$: no ideal MHD gravitational instabilities.

- The modes are then described by the resistive MHD equations in the following form:

$$\begin{aligned} &\eta\,[\,\lambda^2(\xi'' - k^2\xi) - (F''/\rho)\,Q\,] + \lambda(F/\rho)\,(Q - F\xi) = 0\,, \\ &\eta\,(Q'' - k^2 Q) - \lambda\,(Q - F\xi) = 0\,, \end{aligned} \tag{35}$$

which, in the limit $\eta = 0$, transform into the ideal MHD equations

$$[\,(\lambda^2 + F^2/\rho)\,\xi'\,]' - k^2(\lambda^2 + F^2/\rho)\,\xi = 0\,, \qquad Q = F\xi\,. \tag{36}$$

Note that all terms in these equations are real now.

- First, make everything *dimensionless* by exploiting slab thickness a, density ρ, and magnitude of magnetic field somewhere: $B_0 \;\Rightarrow\; v_A \equiv B_0/\sqrt{\rho}$, so that $\tau_A \equiv a/v_A$.

Tearing analysis (cont'd)

- Next, define *dimensionless parameters* so that one can make various assumptions on the smallness of those parameters to exploit them in *asymptotic expansions.*

- Horizontal wavelength comparable to transverse size a of the plasma:

$$k_0 a \sim 1 \,, \tag{37}$$

  since tearing modes are *large-scale macroscopic MHD modes* involving small-scale resistive effect in the normal (x), but not in the transverse (y, z) directions.

- Next, *exploit magnetic Reynolds number as an ordering parameter*:

$$(R_m)^{-1} \equiv \eta/(\mu_0 a v_A) \ll 1 \,. \tag{38}$$

  Equilibrium decays on diffusion time scale $\tau_D \gg \tau_A$. Resistive modes grow much faster than the resistive diffusion time, but much slower than the ideal MHD time τ_A:

$$(\tau_D)^{-1} \equiv (R_m)^{-1} v_A/a \ll \lambda \ll v_A/a \equiv (\tau_A)^{-1} \,. \tag{39}$$

  This is possible if we can find modes with a growth rate λ that scales as a broken power of the magnetic Reynolds number: $\lambda \sim (R_m)^{-\nu}\, v_A/a$, where $0 < \nu < 1$. This will turn out to be the case. Since R_m is huge, this provides enough parameter space for the asymptotic analysis.

Boundary layer analysis

- For small η, resistive equations (35) automatically lead to ideal MHD equations (36):

$$Q \approx F\xi + (\eta/\lambda)\,[\,(F\xi)'' - k^2 F\xi\,] \approx [\,1 + \mathcal{O}(\eta/(\lambda a^2))\,]\,F\xi \approx F\xi\,, \tag{40}$$

  where resistive correction is negligible *if ξ is assumed to have $\mathcal{O}(1)$ variations only*.

- From our FFF example, we know that this assumption is not justified if ideal MHD singularities $F = 0$ occur. Then, $\xi_{\rm ideal} \sim 1/x \rightarrow \infty$ while the magnetic field variable Q remains finite. Hence, the resistive terms in Eq. (40) become operative in *a small layer surrounding the ideal MHD singularity* limiting the amplitude of ξ and the related current density. Outside this layer, ideal MHD is appropriate.

- Consequently, three regions occur: *two outer ideal MHD regions* where F is not small, and *an inner resistive layer* surrounding the point $F = 0$. The solutions of the three regions have to be matched so that there should be overlap regions where the resistive as well as the ideal solutions are valid.

- Of course, the singularity $F = 0$ can occur anywhere on the plasma interval, but we will position it at $x = x_0 = 0$ for simplicity.

- *Resistive* (drawn) and *ideal* (dashed) *MHD solutions* at the ideal MHD singularity $F = 0 \;\Rightarrow$

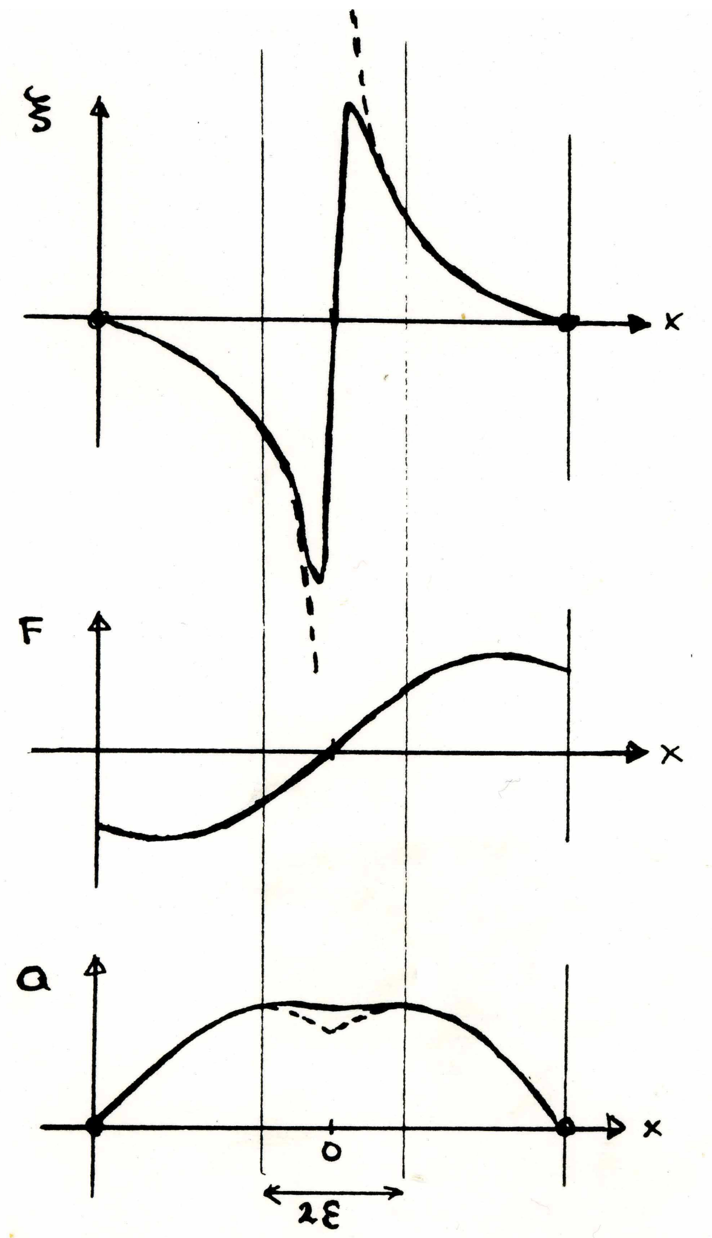
- Solutions can be obtained by either *numerical integration* of the resistive equations over the entire region or by doing an *asymptotic analysis* separating the three regions and matching the solutions at $x = \pm\epsilon$.

- The asymptotic analysis gives explicit expressions for growth rate with *broken powers of magnetic Reynolds number,* justifying the ordering (39).

  [Furth, Killeen & Rosenbluth, Physics of Fluids **6**, 459 (1963)]

  Generalized to resistive internal kinks.

  [Coppi, Galvão, Pellat, Rosenbluth & Rutherford, Sov. J. Plasma Phys. **2**, 533 (1976)]



- Matching involves *jump Δ' of logarithmic derivative* of magnetic field perturbation of outer ideal MHD solution,

$$\Delta' \equiv \frac{a}{Q}\left(\frac{dQ}{dx}\bigg|_{\substack{\text{outer} \\ x\downarrow 0}} - \frac{dQ}{dx}\bigg|_{\substack{\text{outer} \\ x\uparrow 0}} \right), \tag{41}$$

which appears in the explicit expression for *growth rate of the tearing mode:*

$$\lambda = R_m^{-3/5}(KH)^{2/5}(\Delta'/C)^{4/5}\, v_A/a\,, \quad K \equiv ka\,, \quad H \equiv F'(0)a/(kB_0)\,. \tag{42}$$

- This justifies our assumption of broken powers of the magnetic Reynolds number. Estimate of *resistive layer width*:

$$\delta \sim R_m^{-2/5}(KH)^{-2/5}(\Delta'/C)^{1/5}\, a\,, \tag{43}$$

which also conforms to our assumptions.

- Tearing mode analyisis requires $\Delta' > 0$. Example of force free magnetic field gives

$$\Delta' = -2a\sqrt{\alpha^2 - k^2}\,\cot\left(\tfrac{1}{2}a\sqrt{\alpha^2 - k^2}\right), \tag{44}$$

which is positive, when $(\alpha a)^2 - (ka)^2 \equiv H^2 - K^2 > (n\pi)^2$. This agrees with our 'wrong' stability diagram: **The plane force-free field is unstable with respect to long wavelength tearing instabilities, driven by the current!**

Computational methods and extensions

- **Computational methods:** Discretization of the resistive spectral problem (25)–(28) with *Finite Element Method* in direction of inhomogeneity and *Fast Fourier Transforms* in periodic directions, and modern eigenvalue solvers like *Jacobi–Davidson method,*

  [Sleijpen & van der Vorst, SIAM J. Matrix Anal. Appl. **17**, 401 (1996)]

  yield *extremely accurate computer codes computing the complete resistive spectrum* for a given one- or two-dimensional equilibrium, including tokamak or a coronal loop.

  [Kerner, Goedbloed, Huysmans, Poedts & Schwarz, J. Comp. Phys. **142**, 271 (1998)]

- **Extended MHD:** The singular current layers are resolved by replacing (or extending) the Ohmic resistivity term $\eta\mathbf{j}$ with effects of *finite electron inertia* and *the Hall term* of the generalized Ohm's law [see Vol. I, Eq. (3.149)]:

$$-\frac{m_e}{e^2 n_e}\left[\frac{\partial \mathbf{j}}{\partial t} + \nabla\cdot(\mathbf{j}\mathbf{v} + \mathbf{v}\mathbf{j})\right] - \frac{1}{e n_e}\left[\mathbf{j}\times\mathbf{B} - \nabla p_e\right] + \mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta\mathbf{j}\,. \qquad (45)$$

- Presently, intensive research on **magnetic reconnection** starting from extended MHD models, with a wide variety of applications in fusion research (sawtooth crash in tokamaks), space physics (magnetosphere), and astrophysics (stellar flares).

Resistive spectrum: surprise

- Resistivity changes order of system so that the singularities due to vanishing coefficient in front of highest derivative disappear. Hence, one should expect that *the ideal MHD continua split up in discrete modes.*

- This is what happens, but in a totally unexpected way: *multitude of discrete modes on triangular paths* appear in the complex $\lambda \equiv -\mathrm{i}\omega$ plane.

- *Collective effect of ideal MHD continua appears as the damped quasi-mode* inside triangle. This mode is robust: damping remains in the limit $\eta \rightarrow 0$!

  [Poedts & Kerner, PRL **66**, 2871 (1991)]