QUASI-OPTICAL CALCULATIONS OF ECRH POWER DEPOSITION

A.A. BALAKIN, M.A. BALAKINA
Institute of Applied Physics RAS, Nizhny Novgorod, Russia

E. WESTERHOF

Abstract A quasi-optical description of wave beam propagation is described and applied to the calculation of the ECRH power deposition profile in typical tokamak experiments. The quasi-optical model allows a proper description of the effects on beam propagation and absorption from finite spatial and spectral width of the beam as well as from finite spatial and spectral inhomogeneity. Typically, quasi-optical effects result in broadening of the power deposition profile. In case of the ITER Upper Port ECRH launcher this results in about a factor of two reduction of the figure of merit for NTM or sawtooth control as compared to previous estimates.

1. Introduction

In the context of heating, current drive or diagnostics of experiments on magnetically confined plasmas for controlled nuclear fusion, wave propagation and absorption in the electron cyclotron range of frequencies are often said to be well described by geometric optics. In this case, a Gaussian wave beam can be modeled by the propagation of a set of properly chosen individual rays. An exception is formed by focused beams. The description of wave propagation near the beam focus requires the account of diffraction effects. For this purpose several complex geometrical optics models have been formulated, and a number of Gaussian beam tracing codes have been developed. These approaches typically assume an ordering of the different length scales as \( L >> w >> \lambda \), where \( L \) is the scale length on which plasma and wave properties vary, \( w \) is the size across the wave beam, and \( \lambda \) the wave length. Generally, the first of these conditions is broken in the EC resonance layer. A quasi-optical description of wave beam propagation has been proposed assuming only \( L,w << \lambda \) and consequently lifting this flaw of fore mentioned models. The quasi-optical model allows an accurate description of wave beam evolution in the region of EC power deposition including effects from spatial inhomogeneity and dispersion. It describes the aberrations (i.e. deviations from the original Gaussian beam pattern) induced by these effects, whereas existing beam tracing models apply only in the aberration free limit.

Quasi-optically, a broader deposition profile is obtained in the quasi-optical calculations as compared to existing beam tracing codes. Several applications of electron cyclotron heating and current drive, like the control or suppression of plasma instabilities, require optimization of the power deposition or driven current towards the smallest possible profile width. After a brief introduction of the quasi-optical model, this paper discusses the consequences of the additional physics incorporated in the model for the predicted power deposition profile. Of particular importance are the effects of resonance broadening, which is related to the spectral width in the
direction along the magnetic field, and deposition broadening due to spatial inhomogeneity of the absorption, which is related to the beam width along the normal of the EC resonance plane. The importance of these effects is illustrated in dependence of the injection geometry for typical conditions in either ITER or TEXTOR. Finally, the consequences of the additional physics are discussed for the ITER Upper Port ECRH Launcher, whose design is to be optimized for neoclassical tearing mode (NTM) and sawtooth control.9,10

2. A quasi-optical model for wave beam propagation

An electromagnetic wave beam with frequency $\omega$ is considered, and the electric field $E_j(r)$ is written in terms of its scalar amplitude $U(r)$ and operators $\hat{e}_j$. From Maxwell’s equations one then obtains the equation for the evolution of the scalar beam amplitude as

$$\dot{H}[U] = 0, \quad \dot{H} = \hat{e}_j^\dagger (\delta_{ji} \hat{p}_j^2 - \hat{p}_j \hat{p}_j - \delta_{ij}) \hat{e}_j$$

(1)

Where the Hamiltonian operator $\hat{H}$ acting on $U$ contains the dielectric response of the plasma in the form of the operator $\hat{\varepsilon}_\gamma$, and the operator of differentiation, $\hat{p}_j = \hat{\varepsilon}_j / ik_0 \hat{\varepsilon}_\gamma$. The latter is normalized to $k_0 = \omega / c$. Summation over repeated indices is assumed. The dielectric response operator is as yet unspecified. After Fourier transformation, the operator $\hat{H}$ transforms into the eigenvalue $H$ of the dispersion matrix with corresponding eigenvector $e$.

Anticipating the final solution, an expression for the wave field is sought along a reference ray $r_0(\tau)$, where the parameter $\tau$ is introduced acting in the role of time along the reference ray. The reference ray need not be identical to, but could be chosen as the geometric optics ray of the beam axis. Along the ray, an accompanying coordinate system $\{\tau, \xi_1, \xi_2\}$ is defined with $r = r_0(\tau) + g_1(\tau) + g_2(\tau)$, (2)

where in each point of space $g_{1,2}$ and the tangent to the reference ray form an orthogonal set of basis vectors. The Lamé coefficient of the curvi-linear longitudinal coordinate will be denoted as $h$. As in usual geometric optics, the beam amplitude $U$ is now split in a fast oscillating phase and slowly varying amplitude $u$:

$$U = u(\xi, \tau) \cdot \exp(i k_0 \int p_\tau |r_0| d\tau + i k_0 q \cdot \xi),$$

(3)

where $p_\tau$ and $q$ are the normalized longitudinal and transverse wave vectors, respectively, defined on the reference ray. Assuming that the amplitude varies slowly on the scale of a wavelength along the reference ray, substitution of the beam amplitude (3) in the general wave equation (1) finally results in a parabolic equation for the slowly varying amplitude $u$:

$$\frac{\partial u}{\partial \tau} = i k_0 H_0[u] \equiv i k_0 \hat{H} \left(r_0 + \xi, (\hat{r}_0 p_\tau + \hat{q} \xi) \hat{e}_\gamma h + \frac{1}{i k_0 \hat{\varepsilon}_\gamma} \frac{\partial}{\partial \xi} \right) [u],$$

(4)

where the arguments of the Hamiltonian operator $H$ are to be understood as replacing the spatial coordinates, longitudinal, and transverse momenta, respectively, in the original definition of $\hat{H}$.

The next problem one faces, is the specification of the operator representing the dielectric plasma response. In fact, the dielectric response is only known for plane waves as a function in real and wave vector space. For the definition of the complete Hamiltonian operator $\hat{H}$ in terms of space and momentum operators the then obtained from the following Ansatz based on a Taylor-series like expansion:
\[ \hat{H} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{m'=0}^{\infty} \frac{1}{\hat{\xi}^n \hat{\eta}^m \check{x}^n \check{\eta}^m} H \{ \hat{x}^n, \hat{\eta}^m \} \{ \check{x}^n, \check{\eta}^m \} \] (5)

where the bracket is defined as \{ \hat{x}^n, \hat{\eta}^m \} = (1/2)(\hat{x}^n \hat{\eta}^{m'} + \hat{\eta}^m \hat{x}^{n'}), and \( H \) is the usual dispersion function (the appropriate eigenvalue of the dispersion tensor) in real and wave vector space. The main problem for the practical implementation of Eqs. (4) and (5) in a numerical code is formed by the mixed (involving both \( \xi \) and \( q \)) derivatives in (5). Under typical tokamak conditions the main variations of \( H \) are due to spatial inhomogeneity of the magnetic field and dispersion related to the parallel wave vector, i.e. \( H \approx H(\omega, q, a) \). Consequently, there are two preferred directions: one along the gradient of \( B \), and the other parallel to the field, which are practically orthogonal. This allows to split the operator \( \hat{H} \) into parts. Parts involving no mixed derivatives can be evaluated trivially, while parts containing mixed derivatives are assumed small and retained only to second order. As a result, aberrations depending on only either spatial inhomogeneity or dispersion are included, whereas small mixed aberrations are neglected.

3. Aspects determining the power deposition profile

Some of the main factors determining the EC power deposition profile can be categorized as follows. The first is a pure geometrical optics factor coming from the absorption length along a single ray. The second factor comes from the finite spatial width of the beam in the EC resonance layer, and its projection on the local normal to the flux surfaces. Oblique incidence of the beam on the EC resonance layer generally violates the condition \( L >> w \) for the aberration free approximation. Its main geometrical effect on the power deposition profile, however, is included in an aberration free beam tracing code like TORBEAM by the ad hoc projection of the absorbed power in the EC resonance layer rather than in the plane perpendicular to the central ray-trajectory. A third factor comes from the finite width of the beam spectrum associated to its finite spatial width and phase front curvature. This results in effective resonance broadening, as each spectral component is accompanied by its own absorption length and location. Additional factors come from effects that the spatial and spectral inhomogeneity of the absorption have on the trajectory of the beam in both real as well as wave vector space. Due to the absence of a proper connection between rays in real space and in wave vector space, such effects can also not be properly described by models approximating the wave beam by a system of complex geometric optics rays. Though the first two factors are generally well accounted for in present day ray- or beam tracing codes, this does not hold for the remaining factors. A systematic study is presented starting from various launch geometries each of which illustrates possible effects on the power deposition profile from particular aspects of quasi-optical beam propagation as mentioned above.

The first example presents perpendicular launch in the equatorial plane. This illustrates the consequences of resonance broadening as studied in Ref. [8]. Plasma and beam parameters are chosen similar to Ref. [8], and are typical of TEXTOR ECRH experiments with 2nd harmonic X-mode at 110 GHz: major radius \( R_0 = 1.75 \text{ m} \), minor radius \( a = 0.465 \text{ m} \), magnetic field on axis \( B_0 = 2.274 \text{ T} \), Shafranov shift of 8 cm, edge safety factor \( q_e = 4 \), density \( n_e(r) = 4.0 \times 10^{19} (1-(r/a)^2) \text{ m}^{-3} \), and temperature \( T_{e}(r) = 2 (1-(r/a)^2)^2 \text{ keV} \). Figure 1a shows normalized deposition profiles, predicted by the quasi-optical code, for different values of the beam width, \( w_{\text{beam}} \) defined as the radius of the wave beam at the 1/e level of the electric field. The initial beam width and phase front curvature are chosen such that in all cases the beam is focused near the EC resonance...
layer. In agreement with Ref. [8], for well focused beams (typically, $w_{\text{beam}} < 2$ cm) one finds significant broadening of the deposition profile for different values of the beam width $w_{QO}$ and an outward shift of the position $\rho_{QO}$ (in terms of its centre of mass) as compared to a single ray geometric optics calculation ($w_{GO}$, $\rho_{GO}$). In the present geometry a single ray calculation will give an almost identical deposition profile to an aberration free beam calculation as the beam width is tangential to the flux surfaces.

The second example illustrates effects of spatially inhomogeneous absorption across a wave beam. These effects become important when the beam propagates obliquely to the EC resonance layer in the poloidal plane. Propagation in ITER reference scenario 2 is considered, which is characterized by a major radius $R_0 = 6.2$ m, minor radius $a = 2.01$ m, magnetic field $B_\phi = 5.3$ T, plasma current $I_\text{p} = 15$ MA, electron temperature $T_0 = 25$ keV, and density $n_0 = 1.02 \times 10^{20}$ m$^{-3}$. Figure 2a shows results of quasi-optical calculations for a 140 GHz O-mode beam injected with toroidal and poloidal angles $\phi = 0^\circ$, and $\theta = 20^\circ$ and different values of the beam width. For wide beams this is mainly caused by geometrical factors, while for narrow beams the spatial dispersion of the absorption is the determining factor. Figure 2b shows the broadening of the power deposition as a function of poloidal injection angle ($\theta = 90^\circ$ referring to perpendicular injection) for different values of the beam width. Over the whole range, the best localized deposition is obtained with $w_{\text{beam}} = 2$ cm. More oblique the injection results in larger deposition profile broadening.

Figure 1. Power deposition in case of 110 GHz X-mode perpendicular injection in TEXTOR: (a) shows the deposition profile for different values of the beam width, and (b) shows the normalized deposition width $w_{QO}/w_{GO}$ and shift in the center of mass position $\Delta \rho_{QO}$.

Figure 2. Power deposition in case of 140 GHz O-mode oblique injection in ITER: (a) shows the deposition profile for the launching angles $\phi = 0^\circ$, and $\theta = 20^\circ$ and different values of the beam width, and (b) shows the normalized deposition width $w_{QO}/w_{GO}$ as a function of the poloidal injection angle $\theta$ for various beam widths.
4. Power deposition from the ITER Upper Port Launchers

The main tasks of the ITER Upper Port ECRH Launchers will be the control of neoclassical tearing modes or sawteeth. Both applications require optimization of both the driven current and its localization. In case of NTM stabilization, the relevant figure of merit $\eta_{\text{NTM}}$ scales with the inverse of the deposition profile width, i.e. $\eta_{\text{NTM}} \sim 1/w_q$.9,10 The figure of merit for sawtooth control, determined by the required effect on the evolution of the shear at $q = 1$, scales with the inverse square of the deposition profile, $\eta_{\text{sawteeth}} \sim 1/w_q^2$. A number of calculations have been performed to quantify the additional deposition profile broadening from quasi-optical effects. Again the case of ITER reference scenario 2 is considered. The wave beams have been chosen in accordance with the latest UPL design for ITER (see table 1).10,13 Note, that in the ITER ECRH community the injection angles are specified in terms of $\alpha$ and $\beta$, which are related to the poloidal and toroidal launch angles by:

$$\theta = \arcsin(\cos(\beta)\sin(\alpha)) \quad \text{and} \quad \phi = \arctan(\tan(\beta) / \cos(\alpha))$$

To compare the results with present predictions, the deposition profile widths are compared to TORBEAM predictions. Both for the Upper and Lower Steering Mirror, injection angles have been chosen to obtain power deposition at either the $q = 1.5$ and 2.0 surfaces. In case of the USM also a case for deposition at the $q = 1.0$ surface (at $\rho = 0.4$) has been analyzed. Since in the quasi-optical calculations beam propagation and absorption are calculated self-consistently according to the (weakly relativistic) warm plasma dispersion, the required injection angles differ slightly from those used with TORBEAM, while the latter are consistent with the injection angles given in Ref. [10] for deposition at these rational $q$ surfaces. Figure 3 shows the predicted power deposition at $q = 1.5$ and 2.0 in case of launch from the USM and LSM, respectively. The figures show the results from both the quasi-optical and TORBEAM codes. Also the result from an aberration free calculation is given, which coincides reasonably well with the TORBEAM calculation as long as the launch is not too oblique (case of Fig. 3b). In all other cases the predicted deposition width is considerably larger than the one obtained from TORBEAM.

### Table 1: Parameters of the ITER UPL.9,13

<table>
<thead>
<tr>
<th>Mirror</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
<th>$D_{\text{focus}}$ (m)</th>
<th>$w_{\text{beam}}$ (cm)</th>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSM</td>
<td>6.900</td>
<td>0.000</td>
<td>4.180</td>
<td>1.620</td>
<td>2.1 cm</td>
<td>18°</td>
<td>34° – 56°</td>
</tr>
<tr>
<td>USM</td>
<td>6.846</td>
<td>0.000</td>
<td>4.393</td>
<td>2.134</td>
<td>2.9 cm</td>
<td>20°</td>
<td>44° – 68°</td>
</tr>
</tbody>
</table>

Figure 3. Power deposition in from the ITER UPL: (a) shows deposition profiles at $q=1.5$ for injection from the USM, and (b) deposition profiles at $q=2$ for injection from the LSM. Results from both a quasi-optical and aberration free calculation are compared with results from a TORBEAM calculation.
5. **Summary and Conclusions**

A quasi-optical model has been presented assuming that both the plasma (dispersion) properties and the wave beam vary slowly on the scale of a wavelength: $L_w \ll \lambda$. The model goes well beyond existing beam tracing codes.\(^4,5\) It provides a proper description of effects on beam propagation and power deposition from finite spatial and spectral width of the beam as well as from finite spatial and spectral inhomogeneity. It typically predicts broader power deposition profiles than obtained from previous ray- or beam tracing codes. Effects responsible for this additional broadening include resonance broadening due to the finite beam width,\(^8\) and changes in the beam trajectory in both real and wave vector space caused by inhomogeneous absorption. The additional broadening from quasi-optical effects has profound consequences for the capabilities of the ITER ECRH system. In particular, the present results indicate a reduction of the NTM and sawtooth figures of merit as $\eta_{NTM} \to 0.6$ $\eta_{NTM}$ and $\eta_{sawteeth} \to 0.5$ $\eta_{sawteeth}$, when compared with previous calculations.\(^10\)

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**References**