LECTURE NOTES
Direct Current and Radio-Frequency discharges

NS-CP430M

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0. Introduction and outline

The main subject of these lecture notes are discharges in atomic and molecular gases. These discharges, used in many applications, differ from the plasmas discussed up to now in the course by their deviation from an "ideal plasma". Nevertheless, we will see that some aspects of ideal plasmas remain. This holds especially for the electrostatic shielding. In the discharges considered, the charged particles gain energy from an applied electric field (DC, AC, waves) and use this energy not only to sustain the plasma by ionization, but also for excitation and, if applicable, dissociation of the molecules.
The charged particles usually are a minority in the discharges we will consider. The ionization degree can be as low as 10^{-5} in low-pressure discharges (1–100 Pa) up to 0.1 in high-pressure thermal plasmas (10^4–10^5 Pa). Nevertheless, the Debye length is usually so small that shielding occurs and a quasi-neutral plasma is created.
In these lecture notes we will address the physics of two types of low-pressure discharges, namely direct current (DC) and alternating current discharges driven in the radio-frequency (RF) range, that is, between 10 and 100 MHz.
Low pressure discharges are usually far from equilibrium. The creation of charged particles by ionization is not balanced by recombination, as is the basis of the Saha equation you have seen in the first lecture. Because of the low plasma density, ions and electrons are lost at the electrodes and other walls facing the discharge. One could see the equilibrium as that of a chemical reaction in a solution in which one of the reaction products is not soluble.
Since the ion density is very low, Coulomb collisions are far less frequent than collisions with the background gas atoms or molecules. Ions have a mass that is (almost) equal to that of the atoms/molecules and easily exchange energy. Therefore the ion temperature stays close to the gas temperature. On the other hand, electrons are too light to transfer a significant amount of energy in an elastic collision. The heating of the electrons by the applied electric field can thus cause a significant difference between the ion(=gas) temperature and the electron temperature. So, from this point of view, the plasma is again far from equilibrium. Electrons can lose energy in inelastic collisions, and that is what we use in the applications. Electrons drive the chemistry in plasmachemical applications, by dissociating the monomers of the gas, and in lighting the excitation of the gas yields photons.
When the plasma density exceeds a few percent of the gas density, Coulomb collisions with the ions becomes dominant again, reducing the difference between the ion(=gas) and electron temperature.
Discharge physics has a long history, and this is often reflected in the units that are used. Often the Torr (1 mm Hg) is used for the pressure. For convenience, the next table gives the conversion factors.

	Pa	Bar	Torr
1 Pa	1	10^{-5}	$7.6 \cdot 10^{-3}$
1 Bar	10^5	1	760
1 Torr	131.6	$1.316 \cdot 10^{-3}$	1

Table 0.1: conversion of pressure units.

1. The DC low-pressure discharge

1.1 Introduction

DC glow discharges have been studied extensively in experiments like the one drawn schematically in figure 1.1. Two electrodes are placed in a tube filled with gas and a DC voltage is applied. A variable resistor in the circuit stabilizes the current (as will be explained later).

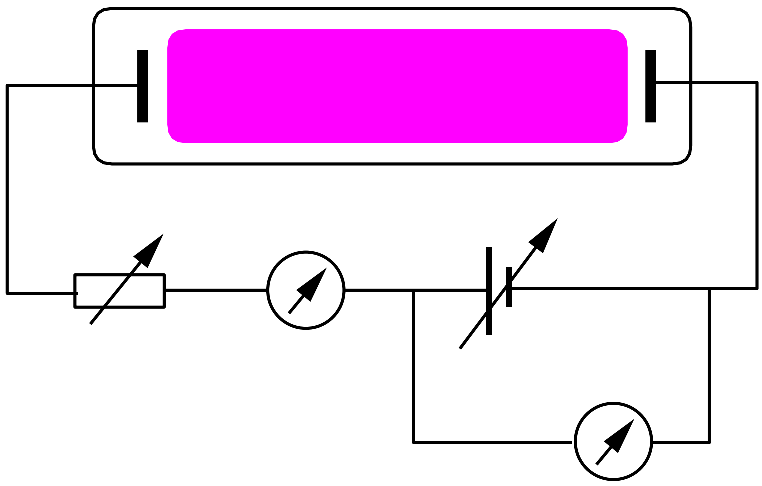

Figure 1.1 A DC discharge with external circuit.

Now let us see what happens when we apply a voltage between the electrodes. At low voltages only incidentally generated (by cosmic radiation, for instance) electrons and ions will arrive at the electrodes and produce a tiny current pulse. The electric field in the gas is constant and equal to -V/d, with d the distance between the electrodes. When the ions reach the electrode, they can release an electron from the surface. From a certain voltage on, these so-called secondary electrons can acquire so much energy from the field that they can ionize the gas and produce new ion-electron pairs. At a sufficiently high voltage this leads to an avalanche and the discharge is ignited. The resistor in the circuit, however, will start to "consume" part of the applied voltage and eventually a situation results in which the discharge is self-sustaining. This kind of discharges is called "Townsend discharge". An important property of a Townsend discharge is that the number of ions and electrons is so low that the electric field is still uniform. (Later on we will see how a large plasma density starts to exhibit shielding, as we would expect from an ideal plasma)
The breakdown voltage depends on the properties of the gas and on the product of the gas density and the distance between the electrodes, N_gd. Usually the temperature is assumed to be fixed and the product pd, with p the pressure, is considered instead. The curve describing the breakdown voltage versus pd is the "Paschen curve". Figure 1.2 shows an example for air. At high values of pd an electron loses so much energy in elastic collisions that it is unable to ionize, unless the voltage is high. At low pd values the mean free path is too large and there are too few ionizations before the electron has crossed the gap between the electrodes. In between there is a minimum breakdown voltage. For a certain setup the Paschencurve also depends on the shape and material of the electrodes. Sharp edges increase the electric field locally and certain materials emit secondary electrons easier.

In order to prevent the generation of plasma in places where it is not wanted, often shielding is used close to an electrode. In that case we are at the left side of the Paschen curve.

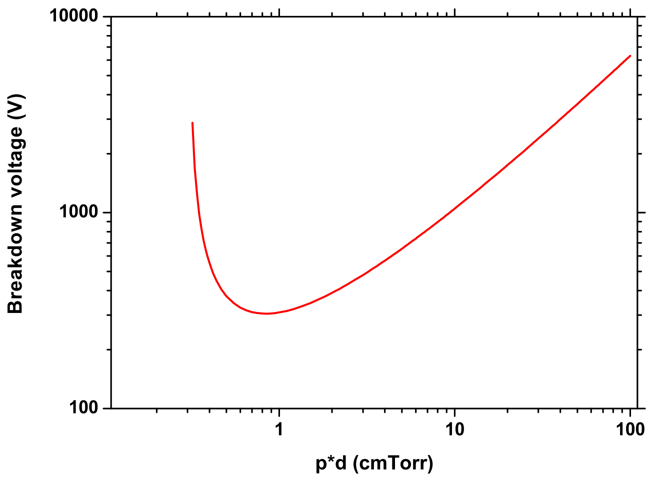

Figure 1.2: Paschen curve for breakdown in air. The secondary electron emission coefficient is set to 10^{-2}.

When the resistance in the circuit is reduced, more current is allowed and more ions and electrons are present in the discharge. The discharge then enters a different regime, the "glow discharge". At even higher currents a transition to an "arc" occurs. This is schematically shown in figure 1.3.

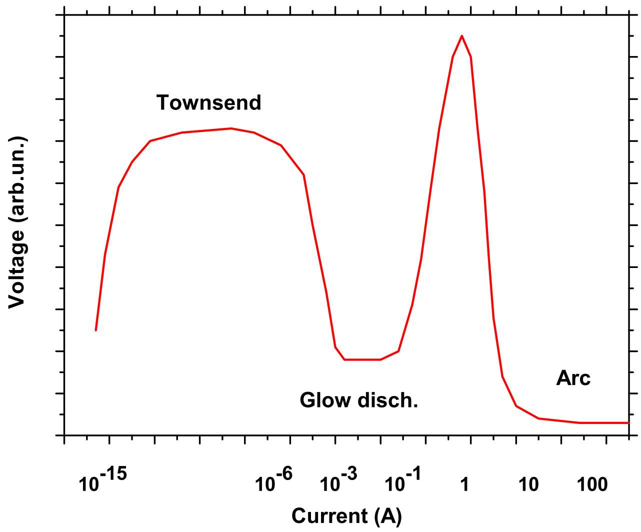

Figure 1.3: Transitions between various kinds of discharges. The current scale is only an indication of the typical range.

The transition from a Townsend discharge to a glow discharge is related to the influence of the generated space charge on the electric field: $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \neq 0$. This makes the field non-uniform, as will become clear in the next paragraphs.
The glow discharge starts with a narrow channel and, when more current has to flow, the channel radius grows. Once the whole electrode is covered, a larger current eventually leads to a much higher degree of ionization and the formation of an arc discharge.

1.2 The Townsend discharge

Let us analyze in more detail what happens in a Townsend discharge. We assume a constant electric field, $E = -V/d$, no end effects, and no gradients perpendicular to the field. The action of the field on the electrons together with the collisions with the background gas will result in a velocity distribution function, $f_e(v)$ and an average drift velocity, w_e. You have met the Boltzmann equation describing this in chapter 6 of the "ideal plasma" lecture notes. The velocity distribution is the same everywhere and results in a source of new electrons and ions, via a rate coefficient k_{ion} (unit $m^3 s^{-1}$):

$$S_{ion} = n_e N_{gas} k_{ion} \quad (m^{-3} s^{-1}) \tag{1.2.1}$$

Over a distance Δx the electron current density, $w_e n_e$, will increase by the amount of ionization:

$$w_e n_e(x + \Delta x) - w_e n_e(x) = n_e N_{gas} k_{ion} \Delta x \tag{1.2.2}$$

With the definition of the so-called first Townsend coefficient α as the number of ionizations per electron per meter:

$$\alpha = N_{gas} k_{ion} / w_e, \tag{1.2.3}$$

we have

$$w_e n_e(x + \Delta x) - w_e n_e(x) = \alpha w_e n_e(x) \Delta x, \quad \text{or} \quad dn_e/dx = \alpha n_e, \tag{1.2.4}$$

so both the density and the current density increase exponentially.
The positive ions move in the opposite direction with a drift velocity w_+, so:

$$w_+ n_+(x) - w_+ n_+(x + \Delta x) = \alpha w_e n_e(x) \Delta x = \alpha w_e n_{e0} e^{\alpha x} \Delta x, \tag{1.2.5}$$

giving

$$n_+(x) = C - \frac{w_e}{w_+} n_{e0} e^{\alpha x} \tag{1.2.6}$$

Now we need the conditions at the electrodes. The constant C can be obtained from the condition that no ions leave the electrode at x=d, so $n_+(d) = 0$; this gives:

$$n_+(x) = \frac{w_e}{w_+} n_{e0} \left(e^{\alpha d} - e^{\alpha x} \right) = \frac{j_{e0}}{w_+} \left(e^{\alpha d} - e^{\alpha x} \right) \tag{1.2.7}$$

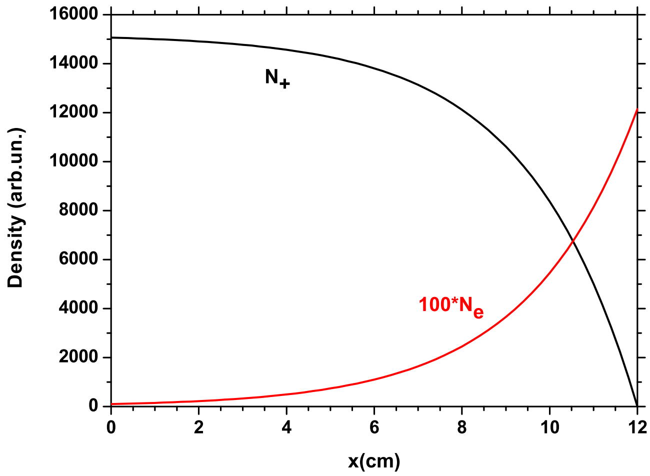

Figure 1.4: Ion and electron density in a Townsend discharge. Note the large difference, resulting in a net positive space charge.

Due to their small mass the electron drift velocity is much larger than the ion velocity. Therefore the ion density is much larger than the electron density in a Townsend discharge. The initial electron current density, j_{e0} follows from the requirement that it is equal to the product of the ion current density and the secondary electron emission coefficient at x=0:

$$j_{e0} = \gamma j_{+0} = \gamma j_{e0}\big(e^{\alpha d} - 1\big), \tag{1.2.8}$$

so

$$\gamma\big(e^{\alpha d} - 1\big) = 1 \tag{1.2.9}$$

Figure 1.4 shows an example of the density profiles in a Townsend discharge. All properties of the gas are present in the Townsend coefficient. Since a multiplication of the gas density, N, with a certain factor and of the electric field with the same factor results in the same average energy change in between two collisions, α and w_e depend on the ratio E/N, and, at the same gas temperature, we have

$$\frac{\alpha}{p} = f(E/p). \tag{1.2.10}$$

An approximate expression for α can be derived as follows. The chance that an electron travels without a collision over a distance larger than "x" is given by Poisson statistics:

$$P(s > x) = \exp(-x/L), \tag{1.2.11}$$

with s the travelled distance and L the mean free path between two collisions. For an ionization the obtained energy over the travelled distance, eEx, should exceed the average ionization energy, written as eV_{ion}, so the chance that an electron will be able to ionize is:

$$P_{ion} = P(s > x_{ion}) = \exp\big(-\frac{V_{ion}}{EL}\big) \qquad (1.2.12)$$

There are on average 1/L mean free paths per meter, so the number of ionizations per electron per meter is the chance that there is an ionization times the average number of mean free paths:

$$\alpha = \frac{1}{L}\exp\big(-\frac{V_{ion}}{EL}\big). \qquad (1.2.13)$$

The mean free path goes as 1/N, or 1/p at constant temperature, so 1/L=Ap, with A a constant:

$$\frac{\alpha}{p} = A\exp\big(-\frac{AV_{ion}}{E/p}\big) = A\exp\big(-\frac{B}{E/p}\big) \qquad (1.2.14)$$

Gas	A($cm^{-1}Torr^{-1}$)	B($Vcm^{-1}Torr^{-1}$)	eV_{ion}(eV)
H_2	5	130	15.4
N_2	12	342	15.5
CO_2	20	466	13.7
H_2O	13	290	12.6
He	3	34	24.5
Ne	4	100	21.5
Ar	14	180	15.7
Kr	17	240	14

Table showing the parameters of the first Townsend coefficient for a number of gases. The approximation is valid for a reduced electric field (E/p) in the range of 100-600 $Vcm^{-1}Torr^{-1}$

The table gives values of A, B, and the ionization energy for a number of gases. Note that the ionization energy used above is higher than the ionization potential of the table, because energy is lost in other collisions as well and the chance of an ionization is not one immediately when an electron reaches the ionization potential.

When we insert the expression for α in the condition for a stable burning discharge, we have

$$\gamma\big(e^{\alpha d} - 1\big) = 1 \qquad \alpha d = \ln\big(1 + \frac{1}{\gamma}\big) \qquad A\exp\Big(-\frac{Bpd}{V}\Big) = \frac{1}{pd}\ln\big(1 + \frac{1}{\gamma}\big), \qquad (1.2.15)$$

or

$$V = \frac{Bpd}{C + \ln(pd)} \qquad C = \ln\Big(\frac{A}{\ln\big(1 + \frac{1}{\gamma}\big)}\Big) \qquad (1.2.16)$$

This is just the Paschencurve. For lower potentials there is no multiplication of electron ion pairs, for higher values we get an avalanche (limited by the circuit).

1.3 The glow discharge and space charge sheaths

When we keep increasing the current density through the DC discharge, the net positive space charge will eventually start to affect the electric field. The discharge then makes a transition to the so-called glow discharge in which the potential drop between the electrodes is concentrated in boundary layers adjacent to the electrodes, while the potential drop over the central region is very small. This is of course related to the shielding of a plasma for disturbances more than one Debye length away. As long as the Debye length is longer than the distance between the electrodes charge separation is possible throughout the discharge, but at higher plasma densities shielding starts to play a role. Suppose we have a semi-infinite plasma bounded by an electrode at a certain potential $V_{el} < 0$, with a vanishing potential at infinity. The plasma has an electron temperature T_e. If there is equilibrium between the force exerted by the electric field (pushing the electrons away from the electrode) and the force by the pressure gradient (due to the higher density away from the electrode), the electron density is distributed according to the so-called Boltzmann factor:

$$en_e\vec{E} = -\vec{\nabla}p_e, \quad \vec{E} = -\vec{\nabla}V, \quad \vec{\nabla}p_e = kT_e\vec{\nabla}n_e \quad \rightarrow \quad n_e(x) = n_{e\infty}e^{eV(x)/kT_e} \tag{1.3.1}$$

Assuming for simplicity that the ion density is constant,

$$n_+(x) = n_{+\infty} = n_{e\infty}, \tag{1.3.2}$$

The potential is given by Poisson's equation:

$$\frac{d^2V(x)}{dx^2} = -\frac{e}{\epsilon_0}n_{e\infty}\Big(1 - e^{eV(x)/kT_e}\Big). \tag{1.3.3}$$

Introducing as a new variable $\Phi = -eV(x)/kT_e$, we obtain:

$$\frac{\epsilon_0 kT_e}{e^2 n_{e\infty}}\frac{d^2\Phi(x)}{dx^2} = 1 - e^{-\Phi(x)}, \tag{1.3.4}$$

or, with $\xi = x/\lambda_D$,

$$\frac{d^2\Phi(\xi)}{d\xi^2} = 1 - e^{-\Phi(\xi)}, \tag{1.3.5}$$

showing that the electrode is shielded and that the characteristic length for the shielding is indeed the Debye length, λ_D. Figure 1.5 shows the solution for $\Phi(0)$=100. The boundary layer is a so-called space charge sheath.

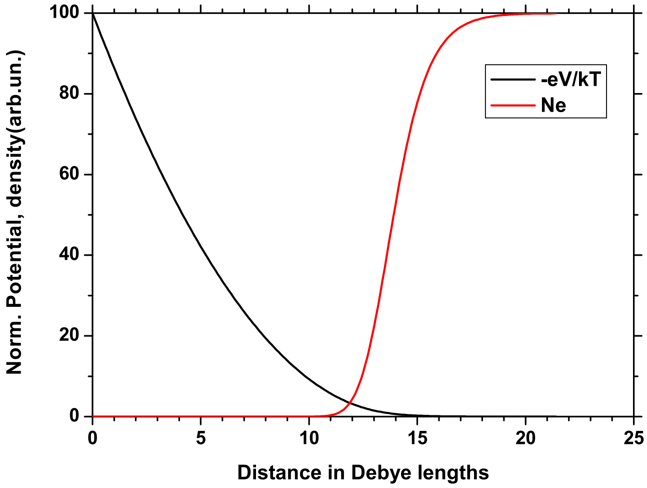

Figure 1.5: Potential in a sheath with a fixed ion density. The boundary conditions are a normalized potental of 100 at the electrode and vanishing potential an E field in the bulk plasma.

So far we have assumed a constant ion density in the plasma. Since there is an electric field in the sheath, the ions are accelerated and the density becomes lower as we approach the electrode. Assuming that there are no collisions in the (usually thin) sheath, the kinetic energy of the ions is given by:

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv_0^2 - eV(x) \qquad v(x) = \sqrt{v_0^2 - \frac{2eV(x)}{M}} \tag{1.3.6}$$

with v_0 the velocity at the entrance of the sheath. The electron density in the sheath is very low, so ionization can be neglected and the flux of ions is constant, $n_+(x)v(x) = n_{+0}v_0$, giving a density:

$$n_+(x) = \frac{n_{+0}v_0}{\sqrt{v_0^2 - \frac{2eV(x)}{M}}}. \tag{1.3.7}$$

With Boltzmann-distributed electrons, and equal electron and ion densities far away from the electrode, the potential is the solution of:

$$\frac{d^2V(x)}{dx^2} = -\frac{e}{\epsilon_0}\Big(\frac{1}{\sqrt{1 - \frac{2eV(x)}{Mv_0^2}}} - e^{eV(x)/kT_e}\Big). \tag{1.3.8}$$

Introducing again the normalized potential $\Phi(x) = -eV(x)/kT_e$ and coordinate, $\xi = x/\lambda_D$, this reads:

$$\Phi'' = \frac{d^2\Phi(\xi)}{d\xi^2} = \frac{1}{\sqrt{1 + \frac{2kT_e}{Mv_0^2}\Phi}} - e^{-\Phi}. \tag{1.3.9}$$

Multiplication of this equation with the first derivative, Φ', and integration from 0 to ξ yields:

$$\int_0^{\xi} \Phi'\Phi'' d\xi' = \int_0^{\xi} \frac{\Phi' d\xi'}{\sqrt{1 + \frac{2kT_e}{Mv_0^2}\Phi}} - e^{-\Phi}\Phi' d\xi'. \tag{1.3.10}$$

Note that ξ runs from zero at the beginning of the sheath to d/λ_D at the electrode, assuming a sheath thickness d. In the plasma, at ξ=0 we take as boundary condition $\Phi(0)$=0, so

$$\frac{1}{2}\left(\Phi'^2 - \Phi_0'^2\right) = \frac{Mv_0^2}{kT_e}\left[\sqrt{1 + \frac{2kT_e}{Mv_0^2}\Phi} - 1\right] + e^{-\Phi} - 1. \tag{1.3.11}$$

Assuming that the electric field vanishes in the plasma, $\Phi'(0)$=0, there is only a real solution when the right hand side of this equation is positive. If we look at the plasma-sheath transition, Φ is small, and we can expand the right hand side in a Taylor series:

$$\frac{Mv_0^2}{kT_e}\left[1 + \frac{kT_e}{Mv_0^2}\Phi - \frac{1}{2}\left(\frac{kT_e}{Mv_0^2}\right)^2\Phi^2 + \cdots - 1\right] + 1 - \Phi + \ \Phi^2 + \cdots - 1 > 0$$

or

$$\frac{1}{2}\Phi^2\left(1 - \frac{kT_e}{Mv_0^2}\right) > 0 \qquad \rightarrow \qquad Mv_0^2 > kT_e. \tag{1.3.12}$$

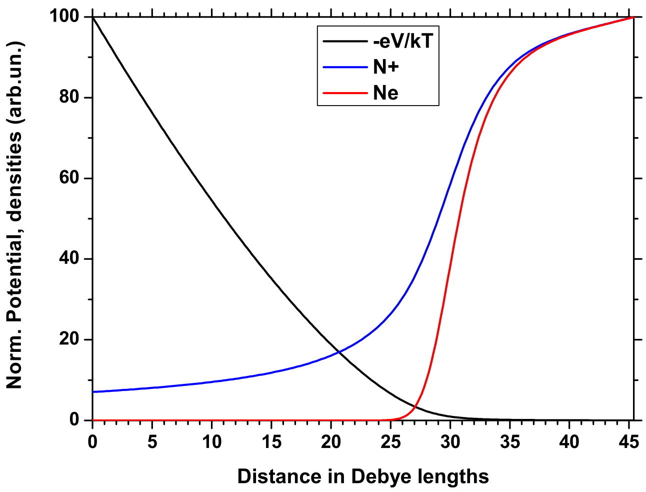

Figure 1.6: Solution for the potential, the ion density, and the electron density in a space charge sheath with an electrode at a normalized potential 100 and ions fulfilling exactly the Bohm criterion. Note that the electrode is at the left side.

This condition is called the Bohm criterion. The ions must already have acquired a certain energy before they enter the sheath. This implies that the quasi-neutral plasma is not at the same potential everywhere. Prior to the sheath there is a zone, the pre-sheath, where the electron and ion density are still equal, but where a potential difference of $0.5\mathrm{kT}_e/\mathrm{e}$ is built up. Since the electrons are distributed according to the Boltzmann formula, this means that the electron (and ion!) density is a factor $\mathrm{e}^{-0.5}$ $\approx$0.6 less than the density far away from the electrode. We will encounter this factor a number of times later on. Figure 1.6 shows the result of a calculation with Φ=100 at the electrode and ions fulfilling exactly the Bohm criterion.

1.4. Current characteristic of a space charge sheath

Assuming a Maxwell-Boltzmann distribution function for the electrons:

$$dn(\vec{v}) = f(\vec{v})d\vec{v} = n_{e0}\exp\Big(\frac{eV}{kT_e}\Big)\Big[\frac{m_e}{2\pi kT_e}\Big]^{\frac{3}{2}}\exp\Big(-\frac{mv^2}{2kT_e}\Big)v^2dv\sin\theta d\theta d\phi. \qquad (1.4.1)$$

The current density perpendicular to the surface of an electrode facing a semi-infinite plasma as before and kept at the a potential, V_{el} is:

$$\begin{aligned} j_{e\perp} &= -en_{e0}\exp\Big(\frac{eV_{el}}{kT_e}\Big)\Big[\frac{m_e}{2\pi kT_e}\Big]^{\frac{3}{2}}\int_0^{2\pi}d\phi\int_0^{\frac{\pi}{2}}\int_0^{\infty}v\cos\theta\exp\Big(-\frac{mv^2}{2kT_e}\Big)v^2dv\sin\theta d\theta \\ &= -e\frac{n_{e0}}{4}\sqrt{\frac{8kT_e}{\pi m_e}}\exp\Big(\frac{eV_{el}}{kT_e}\Big) \end{aligned}. \qquad (1.4.2)$$

The ion current density is given by the Bohm value:

$$j_{+\perp} = en_{e0}\exp\Big(-\frac{1}{2}\Big)\sqrt{\frac{kT_e}{M_+}}. \qquad (1.4.3)$$

The sum of the currents leads to a current voltage characteristic as depicted in figure 1.7, showing the current density normalized to the Bohm value. At large negative values the electron current is suppressed, while already for small negative values the current is dominated by electrons and increases exponentially. The current goes through zero at the so-called "floating potential". This is the potential an object immersed in a plasma will obtain. At floating potential the condition of zero current density gives:

$$\frac{eV_{fl}}{kT_e} = \ln\Big[2.4\sqrt{\frac{\pi m_e}{8M_+}}\Big]. \qquad (1.4.4)$$

For argon (M=40M$_H$) $V_{fl} \approx -5.2kT_e/e$.

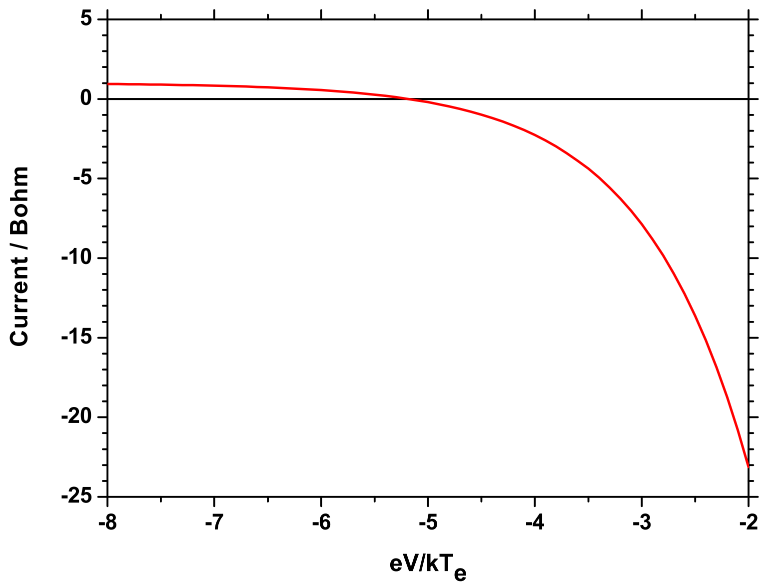

Figure 1.7: The I-V characteristic for a space charge sheath in Argon. The current density is normalized to the Bohm value.

In a DC discharge we have two electrodes, so two sheaths. The current through these sheaths must be equal. Suppose we have one electrode grounded, one electrode at a negative potential, and in between the discharge where shielding causes the electric field to be much lower than in the sheaths, so we may approximate the potential with a constant value, V_{plasma}, see figure 1.8.

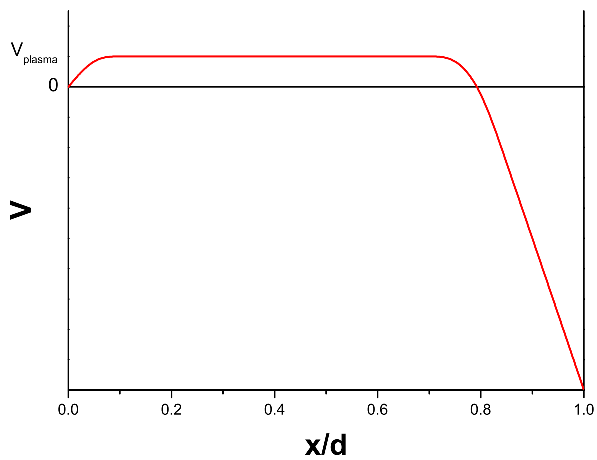

Figure 1.8: The potential distribution in a DC discharge consists of two sheaths and a shielded bulk plasma.

The plasma potential will establish itself such that the net current through both sheaths is equal. At the grounded electrode we will have an electron current that exceeds the ion Bohm current, at the negative biased electrode we will have almost only the Bohm current. Let us work this out in more detail. Neglecting the electron part, the current to the right (negative) electrode is:

$$j_{right} = en_{ep} \exp\Big(-\frac{1}{2}\Big)\sqrt{\frac{kT_e}{M_+}}. \qquad (1.4.5a)$$

and to the right (grounded) electrode:

$$j_{left} = -en_{ep} \exp\Big(-\frac{1}{2}\Big)\sqrt{\frac{kT_e}{M_+}} + e\frac{n_{ep}}{4}\sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{-eV_{plasma}}{kT_e}\Big). \qquad (1.4.5b)$$

Making them equal gives the condition for the plasma potential:

$$e\frac{n_{ep}}{4}\sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{-eV_{plasma}}{kT_e}\Big) = 2en_{ep} \exp\Big(-\frac{1}{2}\Big)\sqrt{\frac{kT_e}{M_+}}, \qquad (1.4.5c)$$

or,

$$V_{plasma} = \frac{kT_e}{2e}\big(1 - \ln(\frac{8\pi m_e}{M_+})\big). \qquad (1.4.5d)$$

For a discharge in Argon this yields $V_{plasma} \approx 4.5kT_e/e$. The net electron current gathered by the grounded electrode are transported via the circuit to the powered electrode, where they are "consumed" by the ions recombining at the surface.

2. The Radio-frequency discharge

2.0 Introduction

When the electrodes are covered with non-conductive material, DC discharges cannot be used. Charging of the electrode surface will generate a counteracting potential and no current will flow in equilibrium. Therefore, one often turns to capacitively coupled radio-frequency discharges. The capacitance in the circuit prohibits a DC current to run through the substrate, thus limiting heating and destruction of, for instance, the connections between components on the chip that is processed.

A commonly used frequency for RF discharges is 13.56 MHz (a frequency that does not interfere with radio communication) and in VHF discharges frequencies up to more than 100 MHz are used. The frequency must be high enough to ensure that the discharge does not decay upon reversal of the applied voltage.

RF discharges are usually operated at similar conditions as DC discharges. The pressure is low, typically 5 to 150 Pa, the lower end of this range is used for etching, the higher end for deposition. Discharge parameters are roughly a density of 10^{15} to 10^{17} m^{-3} and an electron temperature of 1 to 4 eV. These parameters of course depend on the power dissipated (1 to 300 W, approximately) and on the gas mixture that is used. In many gases negative ions are formed; here we focus on electropositive discharges with singly charged ions. The distance between the planar electrodes in experiments is 1 to 10 cm and the radius is 10 cm, typically. A rough outline of an RF reactor is shown in figure 2.1.

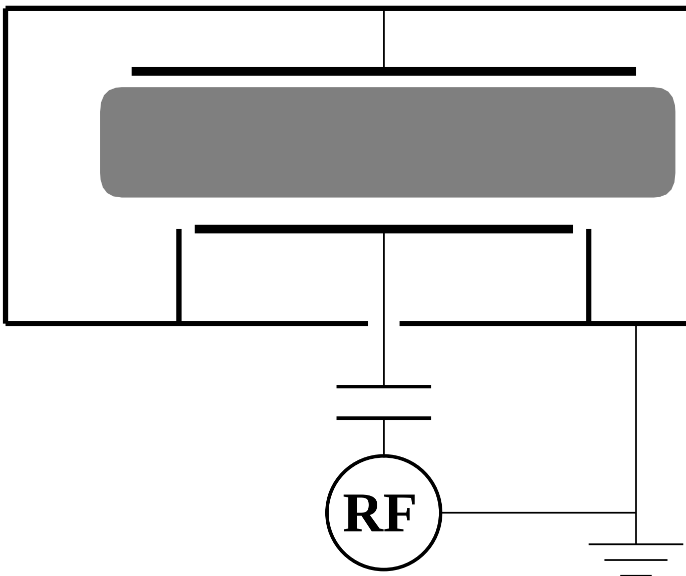

Fig. 2.1. Sketch of the geometry of a planar radio-frequency discharge; not drawn are the gas inlet and vacuum pumps.

2.1. Electrical characteristics.

Similar to the DC discharge, a radio-frequency discharge can be considered as a system of two electrodes immersed in a plasma. Repeating the results of chapter 1.4, the I-V characteristic is given by:

$$I_+ = 0.606 e N_\infty \sqrt{\frac{kT_e}{M_+}} \qquad ; \qquad I_e = -\frac{1}{4} e N_\infty \sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{-e(V_{plasma} - V_{el})}{kT_e}\Big). \quad (2.1.1)$$

The main difference now is that the potential on the powered electrode is harmonic and that the capacitance in the circuit will be charged if there is a net DC current. This charging will provide an additional bias potential that will eventually bring the DC current to zero. The voltage-current characteristic of the planar electrode can be used to describe the electrical features of the RF discharge at low frequencies. Low frequencies in this case means that we do not consider the displacement current generated by the variation of the charge on the electrodes. It is generated by the expansion and shrinking of the sheaths.
Consider again a Maxwellian plasma as before, with two electrodes inserted, at a distance so large that the boundary layers do not overlap and a quasi-neutral plasma with density N_∞ and electron temperature T_e is formed in between. One of the electrodes is grounded, the other one is capacitively coupled to a radio-frequency power source. Since in practice the discharge chamber is also grounded, the grounded electrode will usually have a larger area than the powered electrode.
If we now take V=0 in the quasi-neutral central part (this was V_{plasma} in the DC case, but we can take any reference potential) the current to each of the electrodes (1,2) is given by:

$$I_{1,2}(t) = A_{1,2} e N_\infty \left[0.606 \sqrt{\frac{kT_e}{M_+}} - \frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{eV_{1,2}(t)}{kT_e}\Big)\right], \quad (2.1.2)$$

where A_i is the area of electrode i and V_i its potential difference with the plasma. Note that in general V_1 and V_2 will be negative, because the electrons must be repelled at both electrodes
Since the electrodes are part of a closed circuit (fig.2.4), Kirchoff's law demands that the currents to each electrode are opposite, so:

$$\begin{aligned} A_1 \Big[0.606 \sqrt{\frac{kT_e}{M_+}} - \frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{eV_1(t)}{kT_e}\Big)\Big] &= \\ - A_2 \Big[0.606 \sqrt{\frac{kT_e}{M_+}} - \frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}} \exp\Big(\frac{eV_2(t)}{kT_e}\Big)\Big]&. \quad (2.1.3) \end{aligned}$$

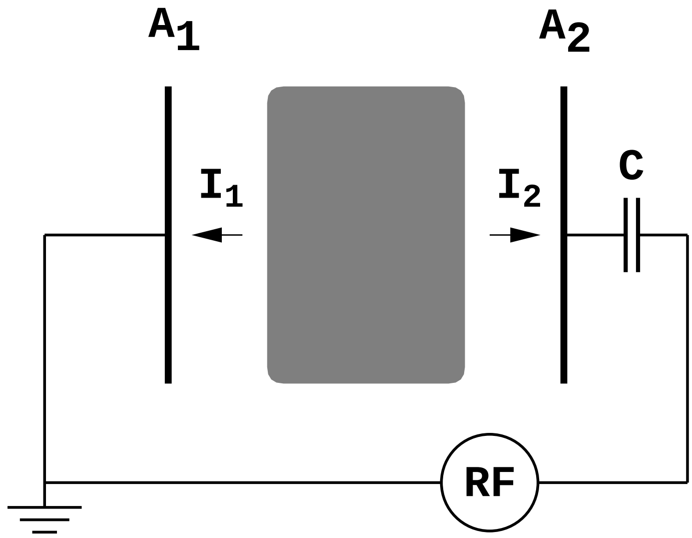

Fig. 2.4 The RF discharge as a system of two planar electrodes.

The capacitive coupling demands that the current averaged over one RF cycle must be zero:

$$\overline{I_{1,2}(t)} = \frac{1}{2\pi}\int_0^{2\pi} I_{1,2} d\omega t = 0. \tag{2.1.4}$$

The potentials, V_2 and V_1 of the electrodes are coupled by the applied RF voltage and possibly a DC bias due to the charging of the capacitor:

$$V_2(t) - V_1(t) = V_0 \sin(\omega t) + V_{DC}. \tag{2.1.5}$$

Let us now introduce the following parameters and normalized quantities:

$$\alpha = A_1/A_2, \qquad \beta = \frac{0.606\sqrt{\frac{kT_e}{M_+}}}{\frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}}} \approx 1.52\sqrt{\frac{m_e}{M_+}}, \qquad \tau = \omega t, \qquad \Phi = -\frac{eV}{kT_e}. \tag{2.1.6}$$

This brings nothing new, but reduces the set of equations to:

$$\alpha\left[\beta - \exp(-\Phi_1(\tau))\right] = -\left[\beta - \exp(-\Phi_2(\tau))\right], \tag{2.1.7}$$

$$\frac{1}{2\pi}\int_0^{2\pi} \exp(-\Phi_1(\tau))d\tau = \frac{1}{2\pi}\int_0^{2\pi} \exp(-\Phi_2(\tau))d\tau = \beta, \tag{2.1.8}$$

$$\Phi_2(\tau) - \Phi_1(\tau) = \Phi_0 \sin(\tau) + \Phi_{DC}. \tag{2.1.9}$$

This cannot be solved without using numerical tools. The procedure leading to a solution starts by choosing Φ_{DC}. With (2.1.9) and (2.1.7) this determines $\Phi_2(\tau)$ and $\Phi_1(\tau)$ for τ running from 0 to 2π. From $\Phi_2(\tau)$ the integral in (2.1.8) can be computed, which should be β when the net current is zero. The value of Φ_{DC} for which indeed the average current vanishes is obtained iteratively, for instance by means of a Newton method.
The results for a reactor with α=2 filled with argon (M_+=40M_H) and Φ_0=120 are presented in figure 2.5. Note, that the discharge switches between two modes, with a very short transient phase in between. Either electrode 1 repels the electrons and I_1 equals the ion current, while the potential of electrode 2 is sufficiently close to zero to have an electron current matching this ion current (I_2=-I_1), or vice versa. Figure 2.6 shows the current densities $I_{1,2}(t)/A_{1,2}$.

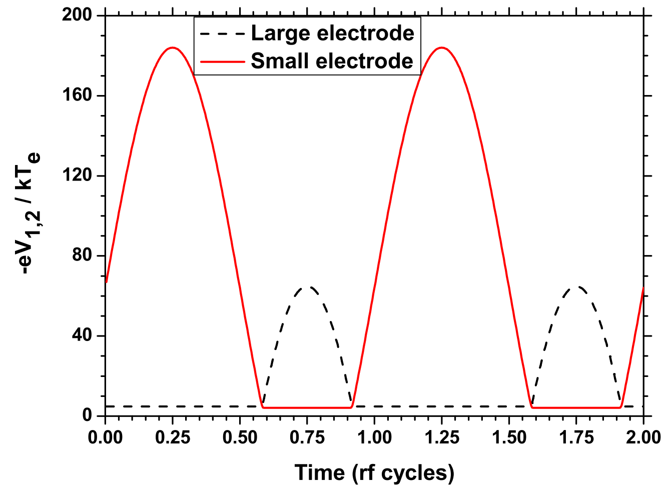

Fig. 2.5 Normalized potential drop over the space charge sheaths in an RF discharge in Argon (M_+=40), with an area ratio α=2, at a frequency of 100 kHz. The normalized RF voltage and DC bias are 120 and 59.7, respectively. Note, that two cycles are shown.

The results obtained so far are valid for low frequencies, where the displacement current does not play a significant role. At high frequencies the displacement current cannot be neglected. Its origin is the variation in time of the charge on the electrodes. This charge depends on the potential difference between the quasi-neutral bulk plasma and the electrode. The net positive space charge in the sheath always equals the negative charge on the electrode, as can be shown using Maxwell's law for a rectangular box with one plane in the bulk (where E=0) and one plane in the (perfectly conducting) electrode (where again E=0), see figure 2.7:

$$\vec{\nabla} \cdot \vec{E} = \frac{e}{\epsilon_0}\Big(N_+ - N_e\Big) \qquad \text{or} \qquad \oint \vec{E} \cdot d\vec{O} = \frac{e}{\epsilon_0} \iiint (N_+ - N_e)\, d\vec{r}, \tag{2.1.10}$$

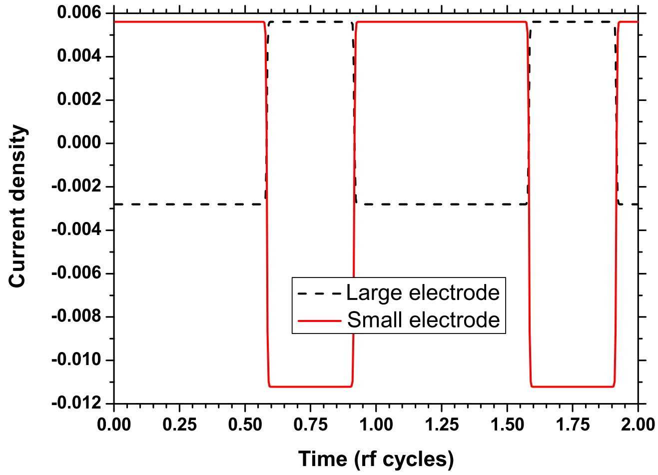

Fig. 2.6 The current density to each electrode for the potential curves of figure 2.5. Again, two cycles are shown.

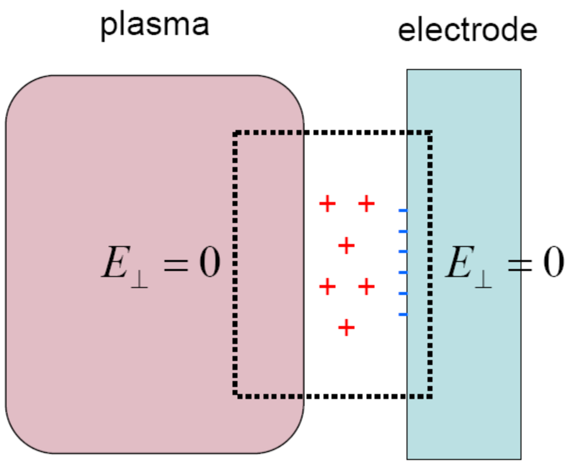

Fig. 2.7 The negative charge on the electrode is equal to the (net) positive charge in the sheath.

To calculate the displacement current we have to specify the positive charge in the sheath. The simplest assumption is, of course, a constant ion density (N_+) and a zero electron density in the sheath and equal ion and electron densities in the bulk. Boundary conditions on the potential are V=0 and E=0 in the bulk (x=0) and V=V_{el} on the electrode, which determines the sheath width, d. Poisson's equation in one dimension,

$$\frac{\partial^2 V}{\partial x^2} = -\frac{eN_+}{\epsilon_0} \tag{2.1.11}$$

then has the solution:

$$V(x) = -\frac{eN_+}{2\epsilon_0}x^2, \qquad d = \sqrt{-\frac{2\epsilon_0 V_{el}}{eN_+}}, \tag{2.1.12}$$

which results in a total charge in the sheath per unit area:

$$Q_{el} = -Q_{sheath} = -eN_+d = -\sqrt{-2e\epsilon_0 N_+ V_{el}}, \tag{2.1.13}$$

and a displacement current, again per unit area:

$$\frac{\partial Q_{el}}{\partial t} = C(V)\frac{\partial V}{\partial t} = \sqrt{-\frac{e\epsilon_0 N_+}{2V_{el}}}\frac{\partial V_{el}}{\partial t} = -\sqrt{\frac{\epsilon_0 N_+ kT_e}{2\Phi_{el}}}\frac{d\Phi_{el}}{dt}. \tag{2.1.14}$$

Thus, the contribution of the displacement current can be considered that of a voltage-dependent capacitance. Since the displacement current depends on the integral of the charge density in the sheath, differences between various sheath models (giving different expressions for C(V_{el}) are not very large. The displacement current has to be added in Kirchoff's law; it does not contribute to the average current since V_{el} is periodic.

Including the displacement current, the current balance to be solved thus becomes:

$$\begin{aligned} A_1\Big[0.606\sqrt{\frac{kT_e}{M_+}} - \frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}}\exp\Big(\frac{eV_1(t)}{kT_e}\Big) - \frac{1}{eN_\infty}\frac{\partial Q_1}{\partial t}\Big] &= \\ - A_2\Big[0.606\sqrt{\frac{kT_e}{M_+}} - \frac{1}{4}\sqrt{\frac{8kT_e}{\pi m_e}}\exp\Big(\frac{eV_2(t)}{kT_e}\Big) &- \frac{1}{eN_\infty}\frac{\partial Q_2}{\partial t}\Big] \end{aligned} \tag{2.1.15}$$

Figure 2.8 shows the influence of the displacement current. All parameters are the same as for figure 2.5, except the frequency, which is increased to 13.56 MHz. The major difference is that the potential on both electrodes has become much more harmonic.

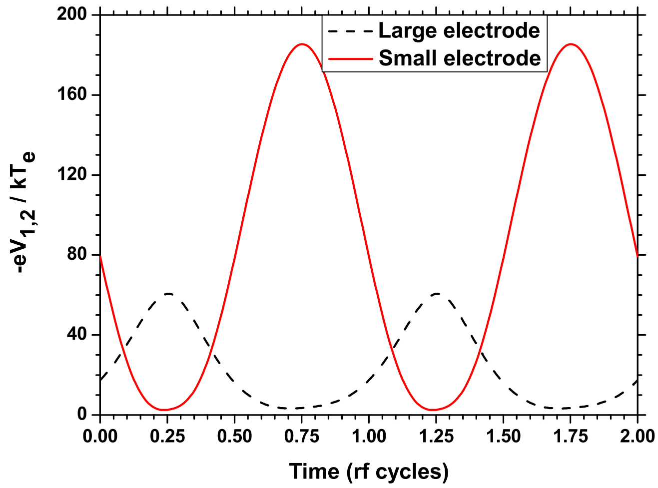

Fig. 2.8 As figure 2.5, but for a frequency of 13.56 MHz. The normalized DC bias is 62.0.

Assumptions made, in the calculation of both figure 2.5 and figure 2.8 are the neglect of secondary electrons (released from the electrodes due to the ion bombardment) and a time-independent ion density in the sheath. The flux of secondary electrons is proportional to the ion flux (1%, typically) and will lead to a slightly different parameter β in eqn. 2.1.7. At low frequencies the ion density and the ion flux may vary in time, but certainly at high frequencies this is a minor correction.

Possibly, larger deviations are caused by the fact that the electron and ion current density are not the same at each part of the electrodes. The discharge may, for instance, contract itself in between the electrodes, leaving a less important contribution to the current balance from other grounded parts, like the cylindrical sidewalls.

With the model discussed, we can compute the behaviour of the bias as a function of the applied RF voltage at a fixed ratio α and as a function of α at a fixed value of V_{RF}. Results are given in figures 2.8 and 2.9, for discharges operated at a frequency of 13.56 MHz.

Obviously, the bias voltage enhanced the potential difference between the plasma and the smallest electrode (powered) and reduces the difference at the grounded electrode. This also enhances/reduces the energy of the ions falling on the electrodes. In an application that requires large ion energies and almost perpendicular velocities, like ion assisted etching, the substrate is therefore attached to the smallest electrode. For deposition the grounded electrode is used.

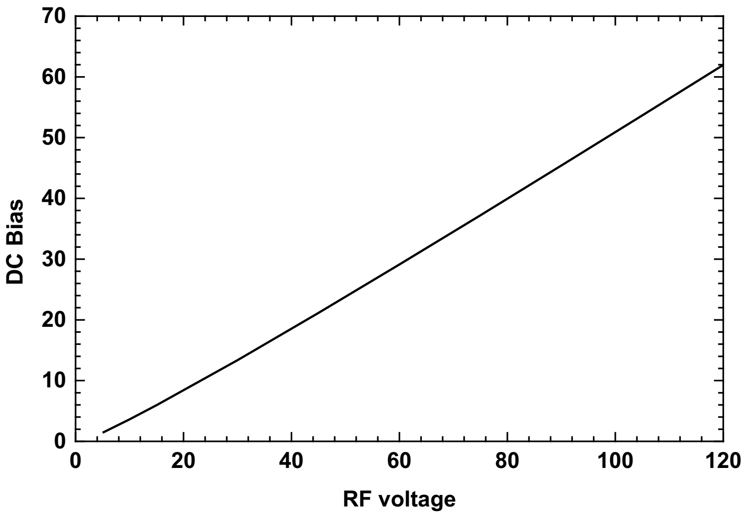

Fig. 2.9 Bias versus RF voltage for an area ratio α=2. The frequency is 13.56 MHz.

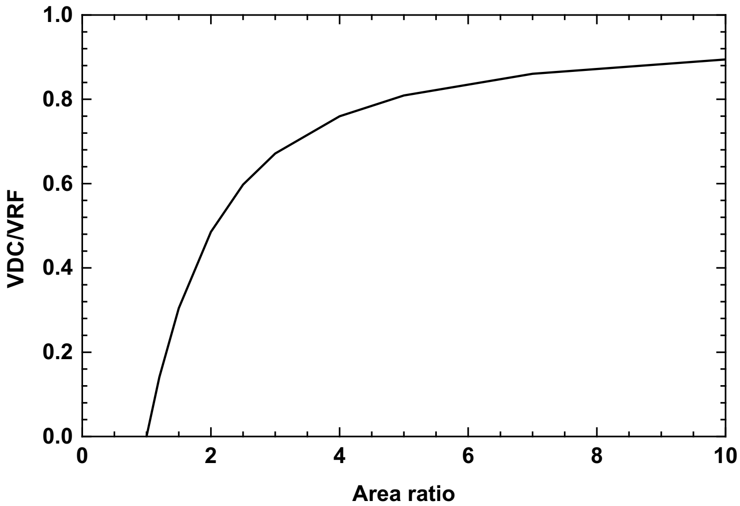

Fig. 2.10 Ratio of bias and RF voltage versus area ratio, for a driving voltage eV_{RF}=120 kT_e. The frequency is 13.56 MHz.