

Mid-term test Plasma Physics, 6 November 2008

Note: The collision time plug-in formulas (36)–(41) on pp. 13–14 of the lecture notes take densities in m^{-3} and temperatures in keV. Some constants:

$$\begin{aligned} m_e &= 9.1094 \times 10^{-31} \text{ kg}, & e &= 1.6022 \times 10^{-19} \text{ J/eV}, \\ m_p &= 1.6726 \times 10^{-27} \text{ kg}, & \mu_0 &= 4\pi \times 10^{-7} \text{ H/m}, \\ k &= 1.3807 \times 10^{-23} \text{ J/K}, & \epsilon_0 &= 8.8542 \times 10^{-12} \text{ F/m}. \end{aligned}$$

1 (40 pt)

A fully ionized hydrogen plasma is immersed in the magnetic field $B = \mu_0 I_z / (2\pi r)$ from an infinite straight wire carrying a current $I_z = 2 \times 10^6 \text{ A}$. This plasma has a uniform density $n = 10^{19} m^{-3}$, and anisotropic temperatures $T_\perp = 2 \text{ keV}$, and $T_\parallel = 5 \text{ keV}$.

- What are the average proton velocities parallel and perpendicular to the magnetic field?
- What are the typical electron and ion gyro-radii at a distance r from the wire?
- Compare the ion gyro-radius with the inhomogeneity of the magnetic field (the length scale on which the field strength changes significantly and the length scale on which the field lines curve). How high must the current I_z be so that the *guiding center approximation*, which says that particle drifts are small corrections to the gyro-motion around a fieldline, is okay for the ions?
- In the guiding center approximation, what are the average ion and electron ∇B and curvature drift velocities at distance r from the wire?
- Express the total current density in terms of the ion and electron drift velocities. Compute the current density. In which direction does this current flow? (ignore the magnetic field due to the current in the plasma)

2 (30 pt)

A plasma is in contact with an infinite (in the y, z directions) conducting plane. Say, the plane is at $x = 0$ and the plasma at $x > 0$. The plane is at an electric potential ϕ_0 compared with the far-away plasma ($x = \infty$). We are looking for the electric potential $\phi(x)$ in the plasma. Assume that the electrons have a Boltzmann distribution $n_e(x) = n_\infty \exp(e\phi/kT_e)$ and the ions, with charge eZ , have a distribution $n_i(x) = (n_\infty/Z) \exp(-eZ\phi/kT_i)$.

(Hint: This problem uses approximations analogous to the case of the Debye-screened electric potential around a point-charge, which has spherical symmetry. Here the symmetry is different: there is only x -dependence, not r -dependence.)

- What is the charge density ρ_E as a function of ϕ ?
- From $\mathbf{E} = -\nabla\phi$ and Poisson's equation for the electric field, derive the (differential) equation that $\phi(x)$ must satisfy in the plasma ($x > 0$).
- Assume that the plasma is ideal. Explain what this means for the typical kinetic and potential energies of the particles, and use this knowledge to approximate the expressions for $n_e(x)$ and $n_i(x)$ so that they are linear functions of ϕ .
- Simplify the differential equation (question a) for $\phi(x)$ accordingly and solve it. Write the solution $\phi(x)$ in terms of the Debye length.

3 (30 pt)

Compute, assuming that the following three plasmas are ordinary hydrogen plasmas,

	n (m^{-3})	T (eV)	B (T)
Solar corona	10^{16}	100	0.03
Fusion reactor	10^{20}	10^4	3

- (a) The plasma frequency and the Debye length
- (b) The average distance between electrons and the distance of nearest approach for two electrons with energies kT . Are these plasmas ideal?
- (c) The electron-ion collision time τ_e and the electron mean free path λ_{mfp} .
- (d) The sound speed c_s (Section 5.1) and the Alfvén wave speed c_A (Section 3.4).
- (e) Consider a 200 kHz Alfvén wave in the fusion plasma. Can the plasma be considered to be collisionless or collisional? Are the ions adiabatic or isothermal?

4 (0 pt)

Write down your preferences for the second part of this course. We have about nine lectures to divide. Think of the following choices:

- Fusion energy / Plasma chemistry / Astrophysics / Complex (e.g. dusty) plasmas
- Theory / experimental methods / applications
- Other ideas