Solution exercise 1cde Plasma Physics, September 17 2009

(1) c Compute the electrostatic energy $U = \int \phi\rho_E \, d^3x$ from expressions (15) and (16) for the electric potential and the charge density.

We integrate over a spherically symmetric potential distribution:

$$U = \int \phi\rho_E \, d^3x = \int_0^\infty \phi\rho_E \, 4\pi r^2 \, dr$$

Using Eq. (15) and (16) we find

$$U = -\frac{q^2}{4\pi\epsilon_0\lambda_D^2} \int_0^\infty e^{-2r/\lambda_D} \, dr = \frac{q^2}{8\pi\epsilon_0\lambda_D}.$$

(d) Show that $\lambda_{mfp}/\lambda_D \sim \tau_e\omega_p$.

Multiply expressions (6) and (8) to obtain $\omega_p\lambda_D = \sqrt{kT/m} \sim v_{th}$. From $\lambda_{mfp} = v_{th}\tau_e$ one sees that $\lambda_{mfp}/\lambda_D \sim \tau_e\omega_p$.

(e) In figure 4, why does the 50% ionization curve get so close to the intersection of the line separating ideal/nonideal plasmas $(N_D = 1)$ and the line separating degenerate/nondegenerate plasmas $(kT = E_F)$.
Hint: Make the very crude assumption that the exponential in Saha's equation dominates the temperature dependence, so that $kT \approx E_i$ for 50% ionization.

Non-degeneracy of the plasma requires $E_{kin} \gg E_F$ (see end paragraph 2.2):

$$\frac{3}{2}kT \gg \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}.$$

The plasma is well-ionized for $E_{kin} > E_i$, or:

$$\frac{3}{2}kT > \frac{e^4 m_e}{2\hbar^2(4\pi\epsilon_0)^2}.$$

Taking the geometric mean of these inequalities one finds $E_{kin} \gg \sqrt{E_F E_i}$ or

$$\frac{3}{2}kT \gg \frac{e^2(3n/\pi)^{1/3}}{8\epsilon_0}$$

which, using Eqs. (8) and (19), leads to the ideal plasma condition $N_D \gg 1$. Thus one sees that a well-ionized plasma for which quantum effects (degeneracy, Fermi-Dirac statistics) are unimportant, automatically has sufficiently high temperature and low density that it is an **ideal plasma**.