Solutions exercise 2, Plasma Physics, September 24, 2009

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A charged particle gyrates in a magnetic field B. A number of effects cause a systematic deviation from this constant periodic motion:

1. **gravitational force $m\mathbf{g} \perp \mathbf{B}$,**

2. **electric field $\mathbf{E} \parallel \mathbf{B}$,**

3. **electric field $\mathbf{E} \perp \mathbf{B}$,**

4. **changing electric field $\partial\mathbf{E}/\partial t \perp \mathbf{B}$,**

5. **gradient of the magnetic field strength $\nabla B \parallel \mathbf{B}$,**

6. **gradient of the magnetic field strength $\nabla B \perp \mathbf{B}$,**

7. **curvature of magnetic field lines $\mathbf{b} \cdot \nabla\mathbf{b}$.**

Assume that these effects are small compared to the cyclotron motion. Answer the following questions for each of the effects listed above.

a. **Give the expressions for the deviation from regular gyro-motion.**

b. **Is the effect a drift velocity or an acceleration?**

c. **Is the effect (velocity or acceleration) directed $\perp$ B or $\parallel$ B?**

d. **Does the effect have the same or opposite signs for ions and electrons?**

e. **Is the effect strongest for the electrons or for the ions?**

f. **Does the effect cause a current density $\mathbf{J} = \sum_\alpha q_\alpha n_\alpha \mathbf{v}_\alpha$? And a plasma flow velocity $\mathbf{U} = \rho_m^{-1} \sum_\alpha m_\alpha n_\alpha \mathbf{v}_\alpha$? Do electrons and ions contribute equally to J and U?**

g. **The acceleration parallel to the magnetic field and the drift velocity perpendicular to B can always be written as**

$$\mathbf{v}_D = \frac{\mathbf{F} \times \mathbf{b}}{qB}, \qquad \frac{dv_\parallel}{dt} = \frac{\mathbf{F} \cdot \mathbf{b}}{m}.$$

Examine the expressions you obtained for the drifts and accelerations in exercise 2a. For all these cases, write down the force which, according to the above equations, would cause the effect.

question:	3a,b.	3c.	3d.	3e.	3f.	4. $\mathbf{F} =$
1.	$\mathbf{v} = (m/qB)\mathbf{g} \times \mathbf{b}$	$\perp \mathbf{B}$	$\sim 1/q$	i	$\mathbf{J}_i$, $\mathbf{U}_i$	$m\mathbf{g}$
2.	$d\mathbf{v}/dt = (q/m)E_\parallel \mathbf{b}$	$\parallel \mathbf{B}$	$\sim q$	e	—	$qE_\parallel \mathbf{b}$
3.	$\mathbf{v} = \mathbf{E} \times \mathbf{b}/B$	$\perp \mathbf{B}$	—	=	$\mathbf{U}_i$	$q\mathbf{E}_\perp$
4.	$\mathbf{v} = (m/qB^2)d\mathbf{E}_\perp dt$	$\perp \mathbf{B}$	$\sim 1/q$	i	$\mathbf{J}_i$, $\mathbf{U}_i$	$-m(d\mathbf{v}_E/dt)$
5.	$d\mathbf{v}/dt = -(v_\perp^2/2B)\mathbf{b}\nabla_\parallel B$	$\parallel \mathbf{B}$	—	(e)	—	$-\mu\mathbf{b}\nabla_\parallel B$
6.	$\mathbf{v} = (mv_\perp^2/2qB^2)\mathbf{b} \times \nabla B$	$\perp \mathbf{B}$	$\sim 1/q$	=	$\mathbf{J}_e$, $\mathbf{J}_i$, $\mathbf{U}_i$	$-\mu\nabla_\perp B$
7.	$\mathbf{v} = (mv_\parallel^2/qB)\mathbf{b} \times (\mathbf{b} \cdot \nabla\mathbf{b})$	$\perp \mathbf{B}$	$\sim 1/q$	=	$\mathbf{J}_e$, $\mathbf{J}_i$, $\mathbf{U}_i$	$-mv_\parallel^2\mathbf{b} \cdot \nabla\mathbf{b}$