

Guiding Center Motion

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Outline:

- Particle Gyration and Guiding Center
- Drift Motion
 - external forces
 - inhomogeneous fields
 - slowly varying fields
- Adiabatic Invariants
- Drift Kinetic Equation

Introduction

Goal: confine a high-T plasma with magnetic fields

→ B curved, inhomogeneous.

- Coulomb interactions between particles:
higher energies lead to smaller deflections
→ weakly collisional plasma.
- Coulomb interaction = long range
→ deviations from free particle orbits are caused by small-angle collisions
with many particles simultaneously (often treated statistically).
- Lorentz force dominates particle motion

This lecture:

particle motion in macroscopic E and B fields generated by:

- external sources
- the plasma particles collectively

but not:

- the effects of microscopic fields of the particles

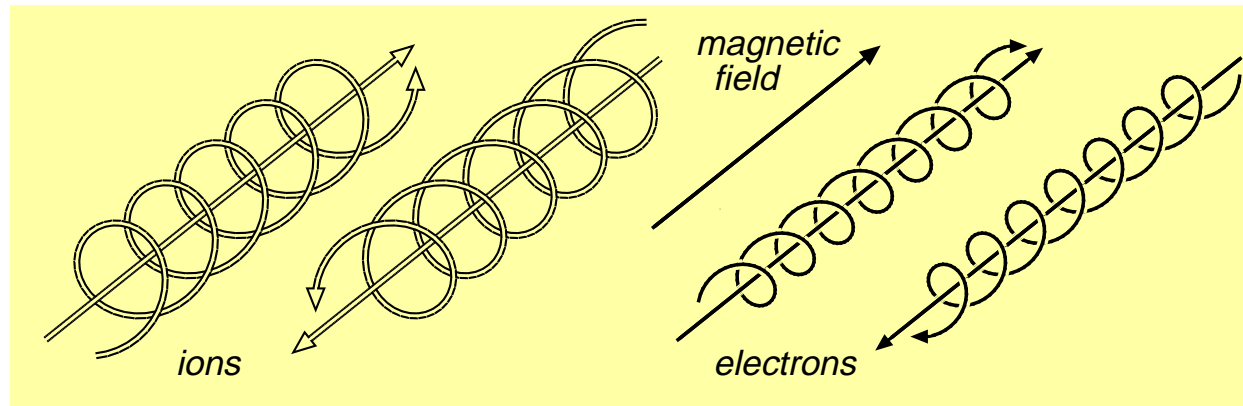
Charged particles in a constant magnetic field

Lorentz force on a particle with electric charge q : $m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B}$.

- Particle energy = constant because $d\mathbf{v}/dt \perp \mathbf{v}$.
- Write $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$ with respect to \mathbf{B} .
- Only \mathbf{v}_{\perp} interacts with $\mathbf{B} \longrightarrow$ circular motion $\perp \mathbf{B}$.

Compute the gyration radius ('Larmor radius'): Centripetal force = Lorentz force

$$\frac{mv_{\perp}^2}{\rho} = qv_{\perp}B \quad \Longrightarrow \quad \rho \equiv \frac{mv_{\perp}}{qB} = \frac{(2mkT)^{1/2}}{qB}.$$



Cyclotron frequency

The frequency of the gyration (cyclotron frequency) ω_c follows from $v_{\perp} = \omega_c \rho$,

$$\omega_c = \frac{qB}{m} \quad (\nu_c = \omega_c/2\pi)$$

- In a typical fusion plasma ($T = 10$ keV, $B = 5$ T):
 - deuteron: $\rho_L = 4.1$ mm, $\nu_{c,D} = 38$ MHz
 - electron: $\rho_L = 67$ μ m $\nu_{c,e} = 140$ GHz($\omega_{c,e}$ comparable to the plasma frequency ω_p).

Magnetic moment

The magnetic moment μ is the product of the current I times the area $\pi\rho^2$ surrounded by the current.

Magnetic moment of a single gyrating particle:

$$\mu = I \cdot \pi\rho^2 = \frac{q\omega_c}{2\pi} \cdot \pi\rho^2 = \frac{mv_{\perp}^2}{2B} = \frac{E_{\perp}}{B},$$

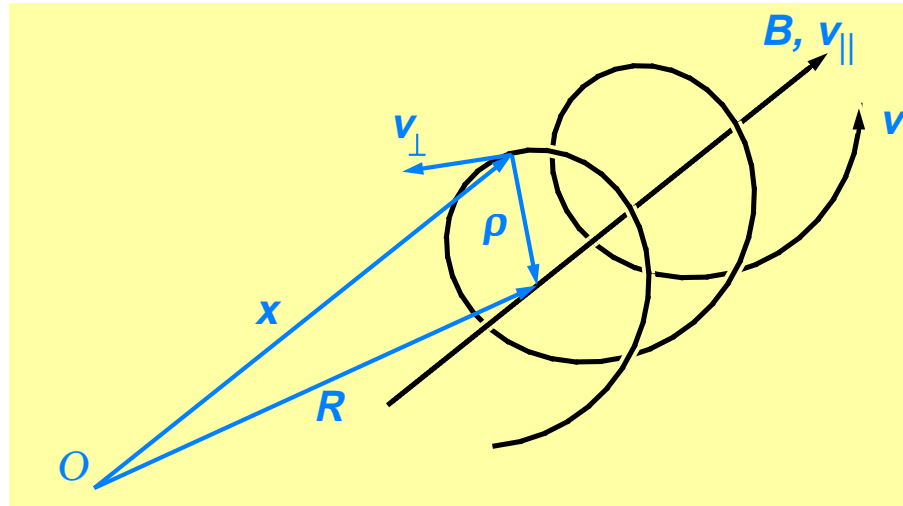
where E_{\perp} is the perpendicular fraction of the kinetic energy.

- The magnetic field produced by the gyrating particles counteracts the externally imposed magnetic field B ,
→ the plasma is **diamagnetic**.

Motion in a constant magnetic field plus an additional force

Acceleration due to Lorentz force plus additional force \mathbf{F} : $m \frac{d\mathbf{v}}{dt} = q (\mathbf{v} \times \mathbf{B}) + \mathbf{F}$.

Separate the motion due to \mathbf{F} from the gyration:



Define the **guiding center** of the particle as

$$\mathbf{R} = \mathbf{x} + \boldsymbol{\rho}.$$

\mathbf{x} is the position of the particle,
 $\boldsymbol{\rho}$ is the gyration radius vector:

$$\boldsymbol{\rho} = \frac{m}{qB^2} \mathbf{v} \times \mathbf{B}.$$

The guiding center motion is

$$\begin{aligned}\mathbf{v}_g &\equiv \frac{d\mathbf{R}}{dt} = \frac{d\mathbf{x}}{dt} + \frac{d\rho}{dt} \\ &= \mathbf{v} + \frac{m}{qB^2} \left(\frac{d\mathbf{v}}{dt} - \frac{d^2\mathbf{R}}{dt^2} \right) \times \mathbf{B} \\ &= \mathbf{v} + \frac{1}{qB^2} \left(q(\mathbf{v} \times \mathbf{B}) + \mathbf{F} - \frac{d^2\mathbf{R}}{dt^2} \right) \times \mathbf{B}.\end{aligned}$$

Using $(\mathbf{v} \times \mathbf{B}) \times \mathbf{B} = -\mathbf{v}_\perp B^2$ we obtain

$$\mathbf{v}_g = \mathbf{v}_\parallel + \frac{1}{qB^2} \left(\mathbf{F} - m \frac{d\mathbf{v}_g}{dt} \right) \times \mathbf{B}.$$

The velocity of the guiding center can be split into components $\perp \mathbf{B}$ and $\parallel \mathbf{B}$,

$$\mathbf{v}_{g,\perp} = \frac{1}{qB^2} \left(\mathbf{F}_\perp - m \frac{d\mathbf{v}_{g,\perp}}{dt} \right) \times \mathbf{B}, \quad \frac{dv_{g,\parallel}}{dt} = \frac{F_\parallel}{m}.$$

- \implies Any force with $\mathbf{F}_\perp \neq 0$ leads to motion $\perp \mathbf{B}$ and $\perp \mathbf{F}$
- $\mathbf{F} = \text{constant} \implies \mathbf{v}_{g,\perp} = \text{constant}$, which leads to the term **drift** for this motion.

E x B drift

A constant electric force $F = qE$ leads to a drift

$$v_E = E \times B / B^2,$$

independent of the charge and mass of the particles.

\implies macroscopic motion of the plasma.

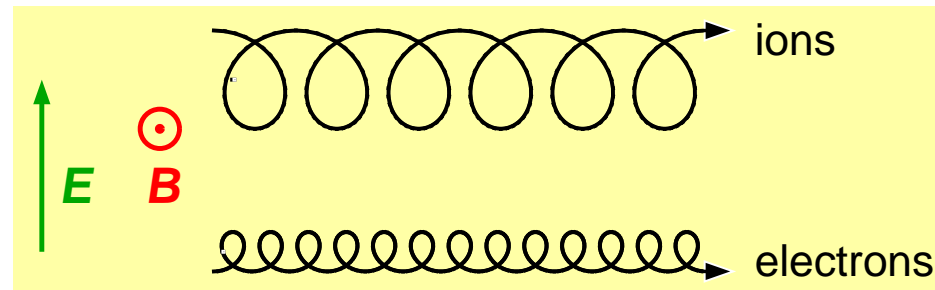


Figure: acceleration / deceleration in E -field

\implies *changing gyro-radius \implies perpendicular drift.*

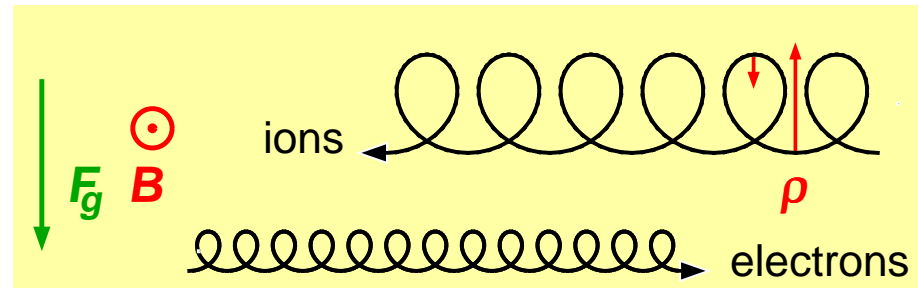
Gravitational force

The drift due to the gravitation force $F_g = mg$ (if $\perp B$) is

$$v_g = mg/qB.$$

Drift direction opposite for electrons and ions \implies
charge separation \implies electric field $\implies \mathbf{E} \times \mathbf{B}$ drift.

Negligible in laboratory plasma: $v_g (B = 5T) \approx 2 \cdot 10^{-8} m/s$



Polarization drift

- B is spatially uniform.
- E depends on time or position: $\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{v}_g \cdot \nabla \mathbf{E}$.
 - \implies the $\mathbf{E} \times \mathbf{B}$ drift is not constant.
 - \implies there is an acceleration $\perp \mathbf{B}$

$$\frac{d\mathbf{v}_E}{dt} = \frac{1}{B^2} \frac{d\mathbf{E}}{dt} \times \mathbf{B}.$$

which gives rise to the **polarization drift**

$$\mathbf{v}_g = \frac{m}{qB^2} \mathbf{B} \times \frac{d\mathbf{v}_E}{dt} = \frac{m}{qB^2} \frac{d\mathbf{E}_\perp}{dt}.$$

This secondary drift depends on charge and mass of the particle. The associated current density is

$$\mathbf{j}_p = \frac{\rho_m}{B^2} \frac{d\mathbf{E}_\perp}{dt},$$

where $\rho_m = m_e n_e + m_i n_i$ is the mass density.

- The electron contribution to \mathbf{j}_p is a factor $O(m_e/m_i)$ smaller than the ion contribution.
- \mathbf{j}_p is a factor c^2/v_A^2 larger than the displacement current.

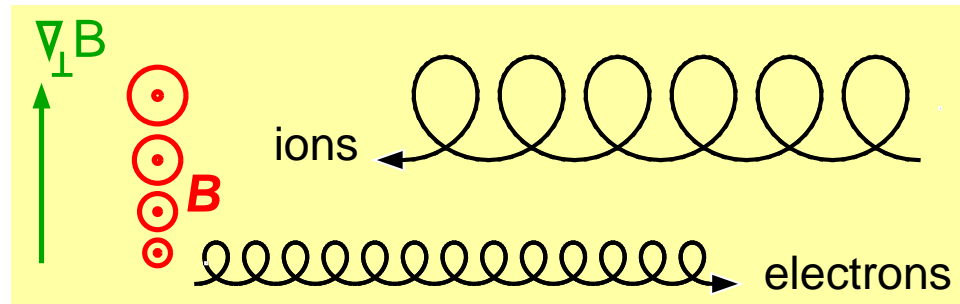
Drifts in inhomogeneous magnetic fields

$$\mathbf{v}_{c,\perp} = \frac{\mathbf{F}_{\perp} \times \mathbf{B}}{qB^2}$$

can still be applied if the relative variation of B is small along one gyration of the particle.

Case 1: Transverse gradient of B

- Graphically, a variation of the gyro-radius over one gyration period.



- Effectively, a particle with magnetic moment μ feels a **force**:

$$\mathbf{F}_{\nabla B} = -\mu \nabla B,$$

leading to a drift

$$\mathbf{v}_{\nabla B} = -\frac{mv_{\perp}^2}{2qB^3} (\nabla_{\perp} B) \times \mathbf{B}.$$

Case 2: Curved magnetic field

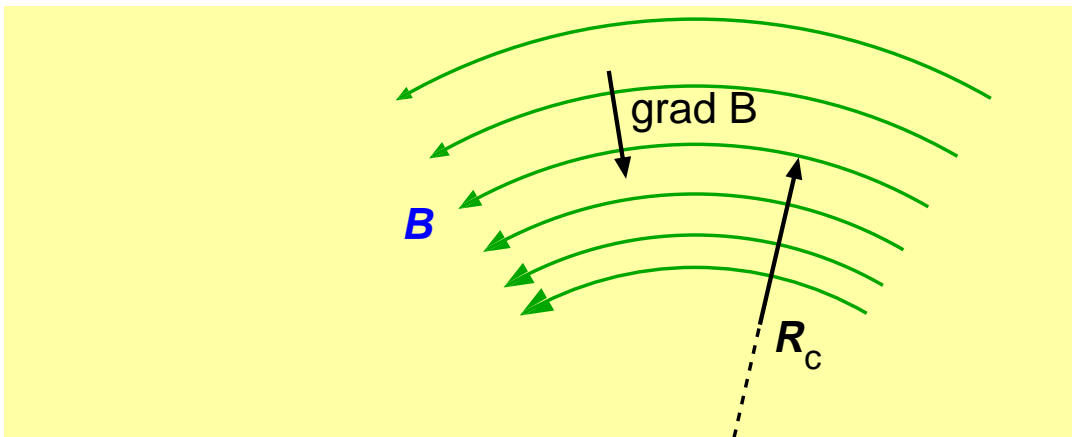
Consider curved magnetic field with curvature radius R_c .

- Particles follow curved field lines \longrightarrow Centrifugal force $mv_{\parallel}^2 R_c/R_c^2$.
 \longrightarrow curvature drift

$$\mathbf{v}_R = \frac{mv_{\parallel}^2}{qB^2} \frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2}$$

- assume: force-free plasma ($\mathbf{j} = \nabla \times \mathbf{B} \parallel \mathbf{B}$).

\implies Relation between **curvature** and **field gradient** $\frac{R_c}{R_c^2} = -\frac{\nabla_{\perp} B}{B}$.



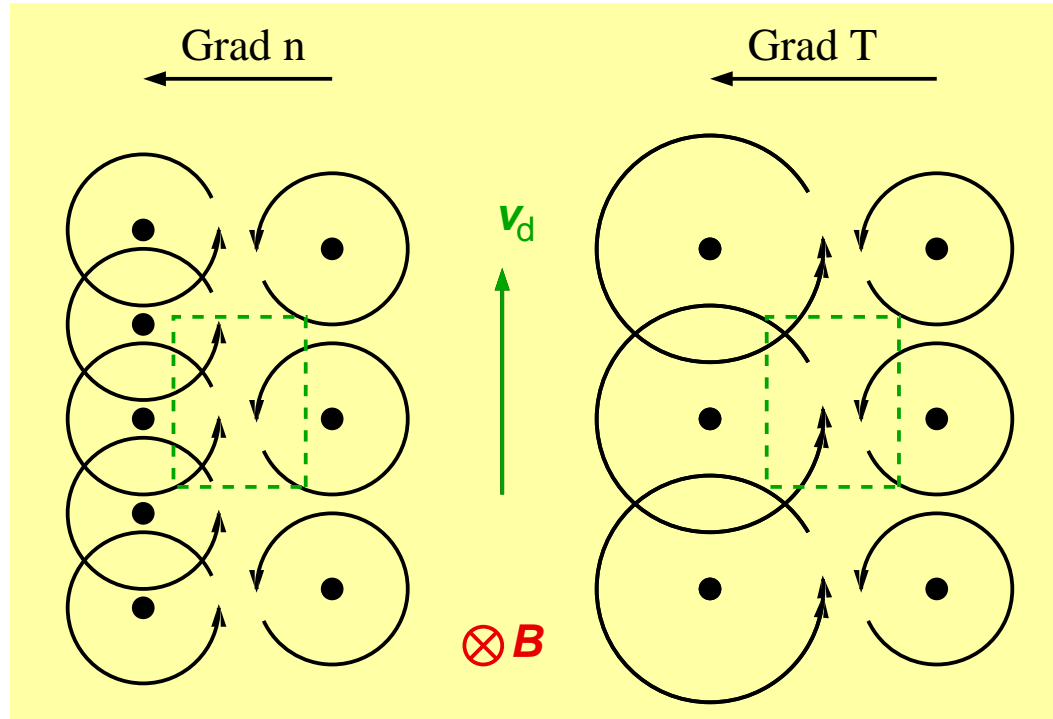
Gradient and curvature of \mathbf{B} combine to $\mathbf{v}_R + \mathbf{v}_{\nabla B} = \frac{m}{qB^3} (v_{\parallel}^2 + \frac{1}{2}v_{\perp}^2) \mathbf{B} \times \nabla B$.

Approximately: $v_{\parallel}^2 + \frac{1}{2}v_{\perp}^2 \approx v^2 \longrightarrow$ resulting drift proportional to the particle energy.
 (Again: drift \longrightarrow charge separation $\longrightarrow \mathbf{E} \times \mathbf{B}$ drift)

Diamagnetic current

The currents $I = q\omega_c/2\pi$ of gyrating particles counteract the magnetic field.

In a plasma, a net diamagnetic current density only exists there where a pressure gradient is present ($\nabla p_e = n_e \nabla T_e + T_e \nabla n_e$, etc.):



From the diamagnetic ion and electron **fluid** velocities,

$$\mathbf{v}_{D,i} = -\frac{\nabla p_i \times \mathbf{B}}{q_i n B^2}, \quad \mathbf{v}_{D,e} = \frac{\nabla p_e \times \mathbf{B}}{e n B^2},$$

sum over all particles \longrightarrow **diamagnetic current**

$$\mathbf{j}_D = n_i q_i \mathbf{v}_{D,i} - n_e e \mathbf{v}_{D,e} = -\frac{\nabla p \times \mathbf{B}}{B^2} \perp \mathbf{B}$$

Plasma diamagnetism

The plasma magnetization is found by summing all particle magnetic moments.

Per unit volume: $\mathbf{M} = -n\langle\mu\rangle\mathbf{b}$.

Thermal plasma: $\langle\frac{1}{2}mv_{\perp}^2\rangle = T \longrightarrow \langle\mu\rangle = T/B$

\longrightarrow Magnetization $\mathbf{M} = -Bp/B^2$.

If $\mathbf{B} = \text{constant}$, the diamagnetic current is

$$\mathbf{j}_D = \nabla \times \mathbf{M} = -\frac{\nabla p \times \mathbf{B}}{B^2}.$$

This current precisely agrees with the force balance in a conducting fluid,

$$\nabla p = \mathbf{j} \times \mathbf{B}.$$

- Note 1: the force per unit volume $\mathbf{j} \times \mathbf{B}$ is the Lorentz force $q\mathbf{v} \times \mathbf{B}$ summed over all particles (using $\mathbf{j} = \sum nq\mathbf{v}$).
- Note 2: ∇B terms in $\nabla \times \mathbf{M}$ cancel ∇B and curvature drifts of the particles.

Adiabatic invariants

For periodic motion, define the action integral $\oint P dQ$ over one period.

- P is a generalized momentum
- Q is the corresponding coordinate

For slow changes of the system (compared with the period), $\oint P dQ$ remains almost constant and is called an **adiabatic invariant**.

First adiabatic invariant

The magnetic moment $\mu = mv_{\perp}^2 / 2B$.

- The periodic motion is the gyration
- P is the angular momentum $mv_{\perp} \rho$
- the coordinate Q is the gyro-angle ϕ .

$$\oint P dQ = \oint mv_{\perp} \rho d\phi = 2\pi \rho mv_{\perp} = 4\pi \frac{m}{q} \mu.$$

Second adiabatic invariant

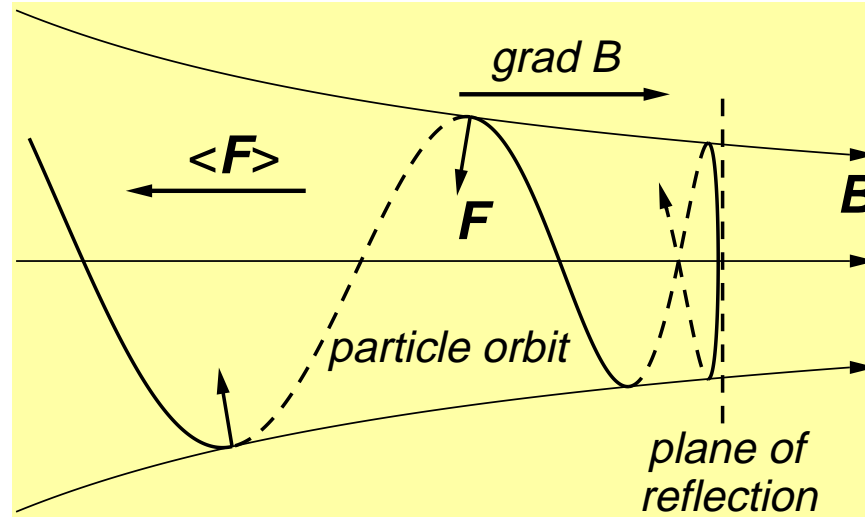
The “longitudinal” invariant $J = \oint mv_{\parallel} dl$.

- Let L be the length between two mirrors.
- $\langle v_{\parallel} \rangle$ is the average longitudinal velocity.

Then $J = 2m\langle v_{\parallel} \rangle L \implies \langle v_{\parallel} \rangle$ increases if L decreases.

- Example: Fermi acceleration principle of cosmic radiation.

conservation of magnetic moment



The figure shows:

- $\nabla_{\parallel} B \neq 0 \implies$ fieldlines are not parallel.
 - \implies the Lorentz force can slow down v_{\parallel} of the guiding center.
 - Lorentz force $\perp \mathbf{B}$ at all times. The shown field lines are envelopes of the particle orbit
 \implies the gyro-orbit always encloses the same number of field lines $\longrightarrow \pi\rho^2 \sim 1/B$.
- $\rho = mv_{\perp}/eB \implies$

$$\mu = \frac{mv_{\perp}^2}{2B} = \text{constant.}$$

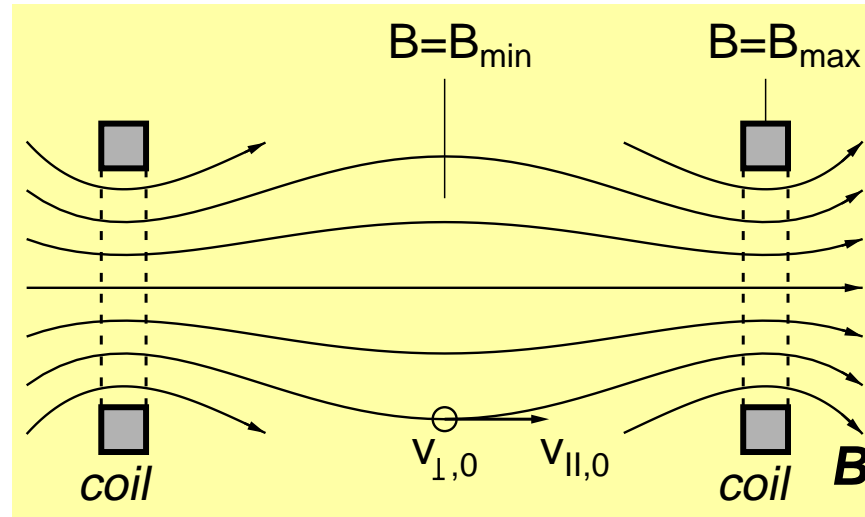
Conservation of energy $\frac{1}{2}mv_{\perp}^2 + \frac{1}{2}mv_{\parallel}^2 = \mu B + \frac{1}{2}mv_{\parallel}^2 = \text{constant} .$

\implies Increasing B leads to

- increased v_{\perp}
- decreased v_{\parallel} .

Magnetic Mirrors

The invariance of μ leads to the mirror principle of magnetic confinement.



Let $v_{\parallel,0}$, $v_{\perp,0}$ be the velocity components at field minimum.

Using energy conservation, the criterion for particle reflection ($v_{\parallel} = 0$) at field maximum is found:

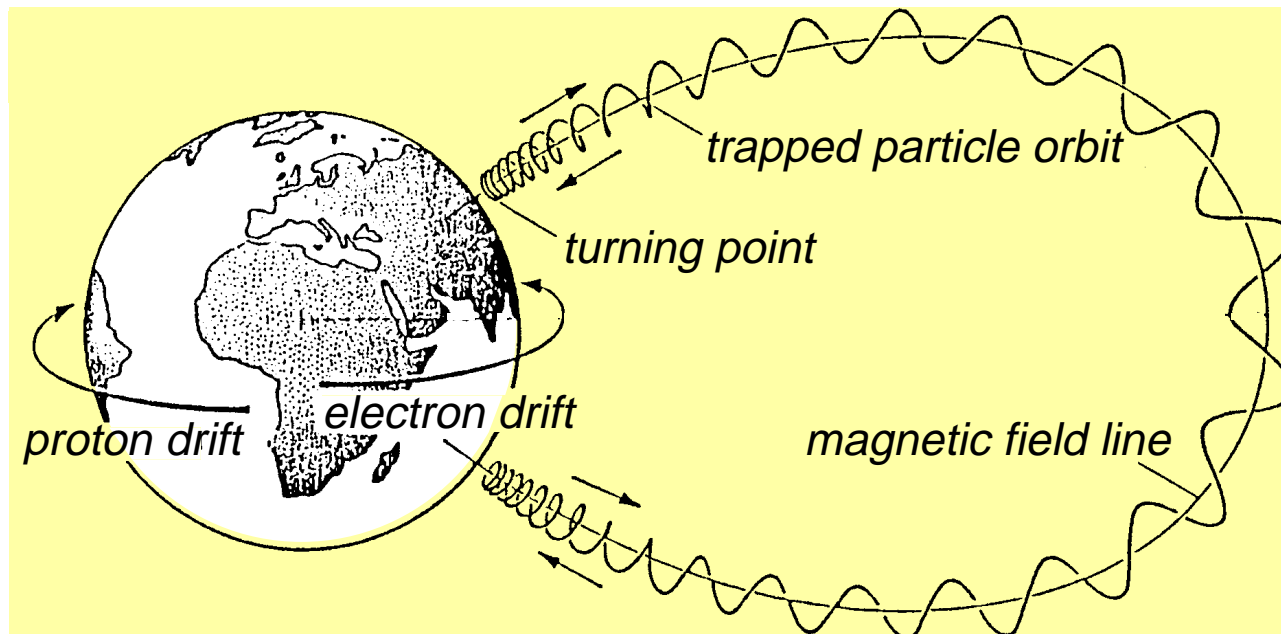
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_{\parallel,0}^2 + \mu B_{\min} \leq \mu B_{\max},$$

Dividing by $\mu B_{\min} = \frac{1}{2}mv_{\perp,0}^2$, one obtains the criterion for particle confinement

$$\frac{v_{\parallel,0}}{v_{\perp,0}} \leq \sqrt{B_{\max}/B_{\min} - 1}$$

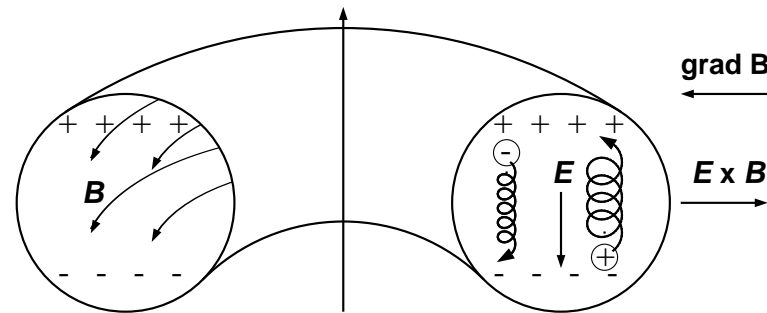
Example: motion in earth magnetic field

- Consider the electrons and protons captured in the earth's magnetic field
- Gradient and curvature of the earth's magnetic field \implies
 - electrons drift from west to east
 - protons in the opposite direction,
- producing the so called 'electron current':



Toroidal systems

The end losses inherent to mirror devices are avoided in the closed geometry of toroidal systems.



A simple toroidal magnetic field is insufficient:

- Field curvature and gradient
- vertical drifts in opposite directions for ions and electrons.
- charge separation
- outward $E \times B$ drift for electrons and ions alike.
- **unstable** plasma configuration.

The magnetofluid equation $\nabla p = j \times B$ says the same.

Additional magnetic field components are needed
(to make a helically twisted field)

1. Complex external coils, as in **stellarators**
2. Toroidal plasma current, as in **tokamaks**

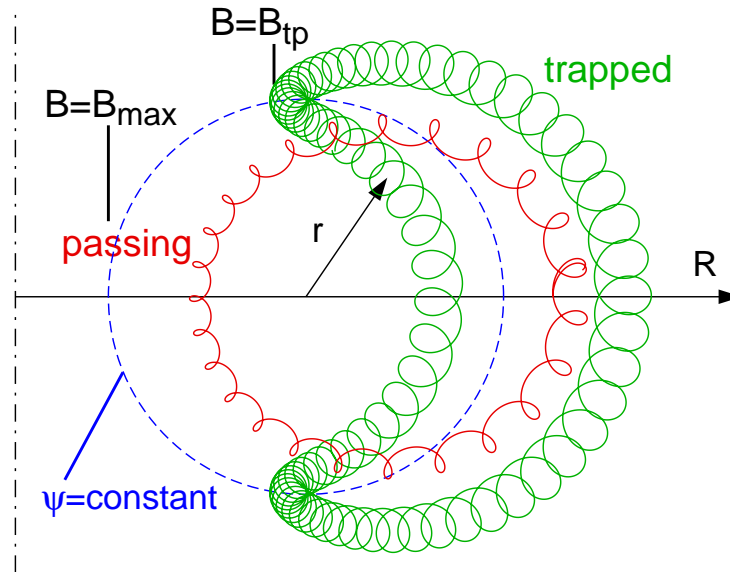
Passing orbits and banana orbits

Poloidal field \longrightarrow closed flux surfaces $R A_{\text{tor}} = \psi = \text{constant}$.

Axisymmetry \longrightarrow conserved canonical momentum

$$P_{\text{tor}} = m v_{\text{tor}} - q A_{\text{tor}} \simeq m v_{\parallel} - q \psi / R = \text{constant}.$$

\longrightarrow poloidally closed orbits.



$\epsilon > \mu B_{\text{max}}$ \longrightarrow particle circulates around the torus.

$\epsilon < \mu B_{\text{max}}$ \longrightarrow particle reflected at the point $\epsilon = \mu B_{\text{tp}}$ (trapped on the low field side).

Banana orbit width:

$$\Delta r = \frac{\Delta \psi}{\partial \psi / \partial r} = 2 \frac{m R v_{\parallel}}{q \partial \psi / \partial r} = 2 \frac{v_{\parallel m}}{q B_{p,m}}$$

Strongly magnetized plasma

Magnetically **confined** plasma:

necessarily **strongly magnetized**, i.e.

- Lorentz force dominates other forces on the particles.
- Gyration dominates the particle motion $\perp \mathbf{B}$.

The processes that perturb the particle orbits, e.g.

- Magnetic field inhomogeneities
- Changes of macroscopic magnetic and electric fields
- collisions ('mean free path', 'collision frequency')

are characterized by

- **length scales** $\gg \rho$,
- **time scales** $\gg 1/\omega_c$.

These facts can be exploited in the case that only the motion of the **guiding centers** is of interest.

- magneto-hydrodynamic models
- particle kinetic models of many plasma instabilities, turbulence, and transport mechanisms.

Slowly varying fields

Let \mathbf{E} and \mathbf{B} vary on length scales ℓ and time-scales τ much longer than the gyro-motion,

$$\frac{\rho}{\ell} = \frac{v_{\perp}}{\omega_c \ell} \ll 1, \quad \frac{1}{\omega_c \tau} \ll 1.$$

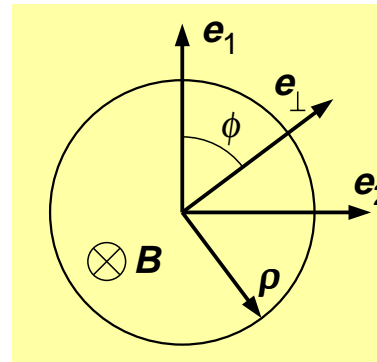
Expand the equations in the **small parameter** $\delta = m/q$: $\rho \sim \delta \ell$, $\tau^{-1} \sim \delta \omega_c$.

The guiding center position is $\mathbf{R}(t) = \mathbf{x}(t) - \boldsymbol{\rho}(t)$, with

$$\boldsymbol{\rho} = \frac{\delta}{B} \mathbf{b} \times \mathbf{u}, \quad \mathbf{u} = \mathbf{v} - \mathbf{v}_E, \quad \mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}.$$

Cylindrical coordinates $(u_{\perp}, \phi, v_{\parallel})$ in \mathbf{u} -space,

$$\mathbf{u} = v_{\parallel} \mathbf{b} + u_{\perp} \mathbf{e}_{\perp},$$



$$\mathbf{e}_{\perp} = \mathbf{e}_1 \cos \phi + \mathbf{e}_2 \sin \phi.$$

The local, orthogonal unit vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{b})$, \mathbf{v}_E , and the field strength B are functions of the particle position $\mathbf{x}(t)$.

We want to give all quantities at the guiding center position $\mathbf{R}(t)$ instead.

Transformation $(\mathbf{x}, \mathbf{v}) \longrightarrow (\mathbf{R}, u_{\perp}, \phi, v_{\parallel})$:

$$\frac{du_{\perp}}{dt} = -\mathbf{e}_{\perp} \cdot \left(v_{\parallel} \frac{d\mathbf{b}}{dt} + \frac{d\mathbf{v}_E}{dt} \right),$$

$$\frac{dv_{\parallel}}{dt} = \frac{1}{\delta} E_{\parallel} + (\mathbf{v}_E + u_{\perp} \mathbf{e}_{\perp}) \cdot \frac{d\mathbf{b}}{dt},$$

$$\frac{d\phi}{dt} = -\frac{B}{\delta} - \mathbf{e}_2 \cdot \frac{d\mathbf{e}_1}{dt} - \frac{1}{u_{\perp}} \mathbf{b} \cdot \mathbf{e}_{\perp} \times \left(v_{\parallel} \frac{d\mathbf{b}}{dt} + \frac{d\mathbf{v}_E}{dt} \right),$$

$$\frac{d\mathbf{R}}{dt} = v_{\parallel} \mathbf{b} + \mathbf{v}_E + \delta \mathbf{u} \times \frac{d\mathbf{b}}{dt} \frac{1}{B} + \frac{\delta}{B} \mathbf{b} \times \frac{d\mathbf{v}_E}{dt},$$

The total time derivative is taken at the particle position,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

In the new variables this becomes

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{d\phi}{dt} \frac{\partial}{\partial \phi} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial}{\partial \mathbf{R}} + \frac{du_{\perp}}{dt} \frac{\partial}{\partial u_{\perp}} + \frac{dv_{\parallel}}{dt} \frac{\partial}{\partial v_{\parallel}}.$$

The only fast-varying quantity is ϕ : $O(\delta^{-1})$. All other derivatives are $O(\delta^0)$.

Further procedure:

- Expand the fields \mathbf{E} , \mathbf{B} , and the unit vectors $e_{1,2}$ in a Taylor series around \mathbf{R} , e.g.,

$$\mathbf{b}(\mathbf{x}, t) = \mathbf{b}(\mathbf{R}, t) + \boldsymbol{\rho} \cdot \nabla \mathbf{b}(\mathbf{R}, t) + \dots$$

(this is a power series in δ)

- Substitute in the equations of motion and keep terms up to the required order.
- Average over the gyro-rotation ($\oint d\phi$)

\implies Guiding center equations for $(\mathbf{R}, u_{\perp}, v_{\parallel})$, independent of ϕ .

\implies All drifts found before, conservation of μ , etc.

The results can be cast into a kinetic equation for the particle distribution function $f(\mathbf{x}, \mathbf{v}, t)$ averaged over ϕ ,

$$\bar{F}(\mathbf{R}, \mu, v_{\parallel}, t)$$

Drift Ordering

Let v_E be of the order of the magnetic drift, i.e., $v_E \ll v_{\parallel}$. The fields vary on the timescale τ set by the drift motion around the system,

$$v_E \sim v_B = O(\delta), \quad \frac{l_{\parallel}}{v_{\parallel}} \ll \tau \sim \frac{l_{\perp}}{v_B}.$$

$$\implies \partial/\partial t, \mathbf{v}_E \cdot \nabla = O(\delta)$$

Introduce new phase variables,

$$\begin{aligned}\bar{\mathbf{R}} &= \mathbf{x} - \boldsymbol{\rho} + O(\delta^2), \\ \bar{v}_{\parallel} &= v_{\parallel} + \delta\mu \mathbf{b} \cdot \nabla \times \mathbf{b} - u_{\perp} \delta I_1(\phi) + O(\delta^2), \\ \bar{\mu} &= \mu - \delta\mu \frac{v_{\parallel}}{B} \mathbf{b} \cdot \nabla \times \mathbf{b} + u_{\perp} \frac{v_{\parallel}}{B} \delta I_1(\phi) + O(\delta^2), \\ \bar{\phi} &= \phi + \delta I_2(\phi) + O(\delta^2).\end{aligned}$$

Here the contributions $I_1(\phi)$ and $I_2(\phi)$ are $O(\delta)$ and periodic in ϕ . They will vanish upon averaging over ϕ .

The guiding center equations are

$$\begin{aligned}\mathbf{v}_g &\equiv \frac{d\mathbf{R}}{dt} = \bar{v}_{\parallel} \mathbf{b} + \mathbf{v}_E + \frac{\delta}{B} \mathbf{b} \times (\bar{\mu} \nabla B + v_{\parallel}^2 \nabla_{\parallel} \mathbf{b}) + O(\delta^2), \\ \frac{d\bar{v}_{\parallel}}{dt} &= \frac{1}{\delta} E_{\parallel} - \bar{\mu} \nabla_{\parallel} B + \bar{v}_{\parallel} \mathbf{v}_g \cdot \nabla_{\parallel} \mathbf{b} + O(\delta^2), \\ \frac{d\bar{\mu}}{dt} &= O(\delta^2).\end{aligned}$$

The last equation shows that the new magnetic moment $\bar{\mu}$ is constant on the time scale on which the fields vary.

Indeed, for the true adiabatic constant we expect changes of the order $\Delta\mu \sim e^{-\omega_c \tau}$. The exponent scales as $-1/\delta \implies \Delta\mu \rightarrow 0$ faster than any power δ^n for $\delta \rightarrow 0$.

The particle energy is

$$\epsilon = \mu B + \frac{1}{2} v_{\parallel}^2 = \bar{\mu} B + \frac{1}{2} \bar{v}_{\parallel}^2 + O(\delta^2).$$

Drift Kinetic Equation

Kinetic equation for the particle distribution function $f(\mathbf{x}, \mathbf{v}, t)$:

$$\frac{df}{dt} = \left(\frac{\partial}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla + \frac{d\mathbf{v}}{dt} \cdot \frac{\partial}{\partial \mathbf{v}} \right) f = C(f).$$

In terms of $(\mathbf{R}, \bar{\mu}, \epsilon, \bar{\phi})$,

$$f(\mathbf{x}, \mathbf{v}, t) = F(\mathbf{R}, \bar{\mu}, \epsilon, \bar{\phi}, t),$$

the kinetic equation becomes

$$\left(\frac{\partial}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial}{\partial \mathbf{R}} + \frac{d\epsilon}{dt} \frac{\partial}{\partial \epsilon} + \frac{d\bar{\phi}}{dt} \frac{\partial}{\partial \bar{\phi}} \right) F = C(F).$$

Note: no $\partial/\partial\bar{\mu}$ -term.

Averaged over $\bar{\phi}$ we find the **drift kinetic equation**

for $\langle F \rangle(\mathbf{R}, \bar{\mu}, \epsilon, t)$:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}_g \cdot \nabla + \left(\bar{\mu} \frac{\partial B}{\partial t} + \frac{q}{m} \mathbf{v}_g \cdot \mathbf{E} \right) \frac{\partial}{\partial \epsilon} \right] \langle F \rangle = \langle C(F) \rangle.$$